Journal of Advanced Sciences and Mathematics Education
Volume 5, Issue 2, 373 - 386
e_ISSN: 2798-2351
DOI: 10.58524/jasme.v5i2.886

Tracing how students make sense of convergent sequences through their preferred mathematical representations: A phenomenological exploration

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Article Info

Article history:

Received: Aug 31, 2025
Revised: Nov 13, 2025
Accepted: Dec 05, 2025

Keywords:

Convergent Sequences;
DMR;
Phenomenology;
Pre-Service Teachers;
Representation;

Abstract

Background: Many students struggle to understand convergent sequences when they depend on only one form of mathematical representation, which limits how they interpret the idea of a sequence approaching its limit.
Aim: This study explores how students who naturally rely on symbolic, visual, or verbal representations experience the process of solving convergent sequence problems. The goal is to understand how they construct meaning, the strategies they choose, and the points at which they feel uncertain when shifting between different modes of representation.
Method: A descriptive phenomenological approach was used with seven participants selected through AHP–TOPSIS classification of Dominant Mathematical Representations. Data were gathered from written work, observations, and individual interviews, then analyzed using Colaizzi's stages. Themes were refined through triangulation to ensure consistency and credibility.
Results: Symbolic-oriented students tended to rely on procedural steps and showed little inclination to move beyond formulas. Students who preferred visual thinking used sketches to build intuition but hesitated when expressing their ideas in symbolic form. Those with a verbal orientation explained their reasoning narratively yet were less confident when formal notation was required. Across all participants, shifts between representations occurred rarely, and emotional responses—such as hesitation or relief—often accompanied these moments.
Conclusion: The findings indicate that students' understanding of convergence is shaped strongly by the representational mode they depend on. This limited flexibility suggests the need for instructional approaches that actively support transitions between symbolic, visual, and verbal representations so students can develop a more connected and meaningful understanding of convergent sequences.

To cite this article: Nursupiamin, N., Rochaminah, S., Pathuddin, P., Sukayasa, S. & Sudarsana, I, W. (2025). Tracing How Students Make Sense of Convergent Sequences Through Their Preferred Mathematical Representations: A Phenomenological Exploration. *Journal of Advanced Sciences and Mathematics Education, 5(*2), 373-386.

INTRODUCTION

The urgency to understand how students make sense of abstract mathematical ideas has become more visible as many still struggle with foundational topics in real analysis (Alam & Mohanty, 2024; Schaathun, 2022). This struggle appears clearly when they encounter convergent sequences, a topic that demands more than routine symbolic manipulation. Students often rely on a single familiar representation, and this habit tends to narrow the way they interpret mathematical behavior (Fiorella, 2023; Schifter & Russell, 2022). When such dependence becomes rigid, their ability to connect formal definitions with intuitive meaning weakens. The difficulty is not always apparent through test scores because assessments often value procedural accuracy over conceptual depth. As

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a result, gaps in understanding persist quietly beneath the surface of correct symbolic work. Educators may assume that mastery of notation signals genuine comprehension, yet the lived experience of students often tells a different story. This disparity motivates the need for a closer examination of how learners actually engage with different forms of representation.

Convergent sequences pose a particular challenge because students must grasp both the formal definition and the intuitive sense of approaching a limit (D'Alessandro & Stevens, 2024; Lamaizi et al., 2024). Many can recite the ε–N definition but struggle to connect it with the behavior of sequence terms as n grows large (Kroeper et al., 2022; Sarker, 2021). This gap illustrates how symbolic expressions, when isolated from other representations, can lose their explanatory power. A sequence may appear straightforward on paper, yet its dynamic nature remains unclear without additional visual or verbal support. Students sometimes treat convergence as a static concept rather than a process unfolding over infinitely many steps. Such misunderstandings reflect deeper difficulties in coordinating multiple ways of thinking. Although real analysis invites students to blend intuition with logic, this blending does not always happen naturally. Understanding why students favor certain representations can clarify where instruction needs to adapt.

The abstract nature of real analysis often makes students feel disconnected from the ideas they are expected to master (Gravett & Winstone, 2022; Pearce, 2023). Unlike more concrete mathematical topics, sequences and limits require learners to imagine patterns that extend beyond visible boundaries (Kokkonen & Schalk, 2021; Tank et al., 2025). When students depend solely on symbols, they may miss the conceptual movement underlying convergence. Visual reasoning might help them form mental images, but not all students feel confident creating or interpreting diagrams. Others who prefer verbal reasoning may understand the general idea yet hesitate when required to translate their thoughts into formal notation. These varying tendencies reveal how representation preferences shape the process of understanding. What appears to be a purely cognitive task is actually intertwined with personal habits of thinking. Recognizing this complexity is essential for making sense of students' difficulties.

Students' experiences with convergence are influenced not only by their cognitive strengths but also by the emotions they bring to the learning process (Acosta-Gonzaga & Ramirez-Arellano, 2021; Wang & Jou, 2023). Some approach symbolic notation with confidence but express discomfort when asked to visualize ideas (Dietrich & Hayes, 2023; Konlan et al., 2021). Others feel relief when they can describe concepts in their own words, yet become anxious when facing formal proofs. These emotional reactions shape how students decide which representations to trust. When a representation feels "safer," students often cling to it even when it limits their understanding. Such patterns reveal how learning is shaped by preference as much as by instruction. The hesitation to shift across representations is therefore not a simple skill gap but a combination of comfort, confidence, and habit. Exploring these experiences allows us to appreciate the subtle factors influencing representational flexibility.

Representations play a central role in shaping how mathematical meaning is constructed, especially in topics where precision and intuition must work together (Barana, 2021; Nathan et al., 2021). Symbolic notation brings structure and rigor, yet it may fail to convey the evolving behavior of a sequence. Visual representations, with their emphasis on movement and approximation, can fill this gap by providing a sense of how terms approach a limit (Peters & Kriegeskorte, 2021; Soleymani et al., 2022). Verbal explanations, meanwhile, allow students to articulate their thinking and make connections across ideas. Each representation supports understanding differently, and none is sufficient on its own. When students over-rely on one mode, their perspective becomes partial and sometimes distorted. This imbalance can hinder the development of a coherent understanding of convergence. Investigating how students engage with these representations helps illuminate why certain misunderstandings endure.

Existing research on student difficulties with convergent sequences has shed light on common errors, but it rarely explores the experiences underlying those errors (Hoth et al., 2022; Kenney & Ntow, 2024). Many studies categorize procedural mistakes without asking why students persist in particular ways of thinking. Similarly, research on mathematical representations often examines performance rather than personal interpretation (Hoth et al., 2022; Satsangi & Sigmon, 2024). These limitations leave unanswered questions about how students internalize representational habits. Without understanding these internal processes, efforts to improve instruction risk addressing symptoms rather than root causes. A phenomenological perspective offers a way to capture the meaning students attach to their chosen representations. By focusing on experience rather than just outcomes, this approach reveals nuances that other methods overlook. This gap in the literature highlights the need for a deeper experiential inquiry.

As mathematics education continues to emphasize representational fluency, understanding how students navigate different modes becomes increasingly important (McNeil et al., 2025; Schulz, 2024). Convergent sequences require coordination between intuition, symbolic accuracy, and conceptual reasoning (Clement, 2022; Luchini et al., 2023). When this coordination falters, students may complete tasks correctly while lacking genuine comprehension. Instruction often assumes that students can shift smoothly from symbols to sketches or from verbal descriptions to notation. In practice, many resist or avoid such transitions, preferring the comfort of familiar representations. This resistance constrains their ability to apply ideas flexibly in new or unfamiliar contexts. Investigating the experiences that shape these tendencies provides valuable insight for designing more responsive teaching approaches. Such insight can help develop instruction that supports smoother transitions across representational forms.

Given these challenges, understanding how students experience the process of solving convergent sequence problems becomes an essential research endeavor (Bakhmat et al., 2023; Yusuf et al., 2023). Their representational choices reveal much about how they interpret concepts, negotiate difficulty, and build meaning (Nielsen et al., 2022; Pham & Tytler, 2022). These choices also signal where they feel secure and where uncertainty arises. Dominant Mathematical Representations offer a useful framework for identifying these tendencies. When combined with phenomenological methods, this framework uncovers layers of experience that traditional assessments cannot capture. Through this perspective, the study seeks to illuminate how students' representational habits shape their learning in subtle yet powerful ways. The insights gained may ultimately guide more effective approaches for helping students grasp fundamental ideas in real analysis.

Research on mathematical representations consistently shows that students who lean too heavily on one mode often face difficulty when working with abstract notions such as convergent sequences, particularly when trying to reconcile formal notation with intuitive mental imagery. Earlier studies tend to document procedural errors but seldom address how these representational preferences shape students' experiences while learning real analysis. The broader mathematical literature also reveals how intricate convergence can be, as seen in the work of Türkmen (2025), who examined robust behaviors in fractional-order operators, and in the investigations of Ibrahim & Çolak (2025) on f-lacunary summable sequences. Similar theoretical depth appears in studies by S. I. Ibrahim et al. (2025), who explored fuzzy-number sequence spaces with Bessel-based formulations. Convergence continues to surface in functional analysis through contributions by Dorai et al. (2025), whose work in Riesz-space approximation highlights the structural richness behind sequence behavior. Applied contexts offer yet another perspective, illustrated by Yang et al. (2025) whose astrophysical models show sequence-like dynamics in stellar evolution. Sequential reasoning also emerges in optimization research by (Schuster, 2025), while cognitive studies by Haase and Hanel link mathematical habits of mind to creative flexibility. The emphasis on multimodal expression appears in STEAM curriculum work by Olivares et al. (2021), reinforcing the value of representational diversity. Complementary insights arise in Luo 2024) analysis of fractional Emden-Fowler equations and in schema-based instruction research by Bowman et al. (2024), each illustrating how coordinated representations support complex reasoning. Despite this wide range of scholarship, there remains little understanding of how pre-service teachers with dominant symbolic, visual, or verbal tendencies personally make sense of convergence, making a phenomenological examination timely and necessary for uncovering the lived realities behind representational dominance.

Understanding how students make sense of convergent sequences requires more than listing the errors they commonly make or identifying which representations they tend to use. Each learner brings habits of thought shaped by prior experiences, comfort levels, and the ways they have learned to approach mathematical ideas. These habits influence how they interpret limits, ε–N definitions, and the behavior of sequences as terms move toward a fixed value. Although earlier studies have classified representational tendencies, such classifications rarely reach the personal and often subtle experiences that arise when students attempt to understand convergence. In classrooms, some students trust symbolic manipulation, while others gravitate toward drawings or verbal explanations, and these choices are often accompanied by moments of doubt, confidence, or confusion that never appear in written assessments. A study that hopes to understand these experiences needs a method that listens to students' voices without forcing them into predetermined categories. A phenomenological approach allows these lived moments—of struggle, clarity, and meaning-making—to emerge naturally, revealing how representational preferences shape students' understanding of convergence.

Even though many studies have documented common mistakes in learning sequences and limits, most remain on the surface of students' observable work and seldom explore what students actually experience while thinking through these concepts. Theoretical contributions from researchers such as Türkmen, Ibrahim, Çolak, Baleanu, Yousif, Alharthi, Mohammed, Dorai, Chil, Wójtowicz, Yang, and Liu illustrate how rich and technically complex the mathematics of convergence can be, yet they do not shed light on how beginners encounter these ideas in the early stages of learning. Research in mathematics education acknowledges the importance of representation, but rarely examines how dominant preferences shape the cognitive and emotional processes students undergo when solving convergent sequence problems. Even studies that map Dominant Mathematical Representations through quantitative methods do not reveal how these profiles influence the meaning students construct while working with abstract ideas. This creates a clear gap: the field lacks an account of how representational tendencies are lived, negotiated, and felt by learners as they confront convergence. Filling this gap is essential for connecting theoretical understanding with authentic student experiences.

The purpose of this study is to explore how pre-service mathematics teachers experience solving convergent sequence problems through the representational mode they naturally rely on—symbolic, visual, or verbal. Rather than starting from a hypothesis that must be tested, the study seeks to uncover the meanings students construct, the strategies they instinctively choose, and the reasons they either remain within or move beyond their preferred representation. Through a descriptive phenomenological approach, the study aims to capture the small but meaningful moments that reveal how students interpret convergence and how representational dominance shapes their understanding. The goal is to build an account that not only describes these experiences but also offers insight into how instruction can better support flexibility and integration across representational forms in real analysis.

METHOD

Research Design

This study was carried out using a descriptive phenomenological approach, as the intention was to understand how students actually live through the experience of solving convergent sequence problems rather than to measure their performance in a numerical sense. Phenomenology allowed the researcher to step back from personal assumptions and pay close attention to how each participant described moments of confusion, insight, or hesitation that emerged during problem solving. This design was selected because representational tendencies—whether symbolic, visual, or verbal—are not merely observable habits but are woven into how students interpret mathematical ideas. By focusing on their lived accounts, the design made it possible to capture nuances that often disappear in more structured or quantitative methods.

Participants

Seven pre-service mathematics teachers took part in this study. Their participation was based on a prior classification using the AHP–TOPSIS model, which helped identify whether they naturally favored symbolic, visual, or verbal forms of representation. The group was intentionally composed of students from different representational profiles to allow a wider span of experiences to emerge during analysis. All students agreed voluntarily to participate in interviews, observations, and written tasks, and each contributed a unique narrative about how they approached the idea of convergence.

Instrument

Three forms of data were collected to gain a fuller picture of each participant's experience. Written tasks provided a direct look at how students attempted to solve convergent sequence problems when left to choose their own representational approach. Observations added another layer, allowing the researcher to notice subtle behaviors such as pauses, gestures, or visual scanning patterns that accompanied their reasoning. Semi-structured interviews offered space for students to talk openly about why they felt drawn to certain representations and what they found difficult or reassuring as they worked through the problems. Together, these instruments created a composite picture of the students' representational experiences.

Data Analysis

Analysis followed Colaizzi's phenomenological method, which emphasizes staying close to participants' own words while gradually moving toward shared themes. The researcher began by reading each transcript and written response several times to become familiar with the tone and texture of the students' experiences. Meaningful statements were then extracted and grouped into clusters that reflected recurring ideas across individuals. These clusters were refined into broader themes that described the essence of how students navigated symbolic, visual, and verbal representations when interpreting convergence. Throughout the process, cross-checking among written tasks, interview accounts, and observational notes helped ensure that interpretations remained grounded in the data rather than imposed by the researcher.

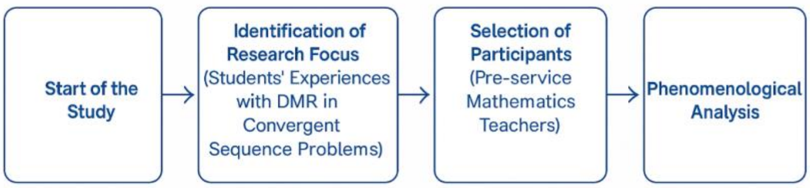

Figure 1. Research Procedure Flowchart

RESULTS AND DISCUSSION

RESULTS

The findings of this study reflect how seven pre-service mathematics teachers experienced the process of making sense of convergent sequences through the representational modes they felt most comfortable using. A descriptive phenomenological analysis was used to interpret their accounts, and the results are presented through several interconnected components: the characteristics of the participants, their dominant representational tendencies, the experiential themes that emerged, and the ways in which participants attempted to shift between representational forms.

Participant Characteristics

The students involved in this study brought different backgrounds and representational preferences, which shaped the way they approached each problem. Their profiles, taken directly from the original document, are shown below.

Table 1. Participant Characteristics

Participant	Gender	DMR	Notes
S01	F	Symbolic	Strong procedural orientation
S02	M	Symbolic	Confident in formal notation
S03	F	Visual	Prefers diagrams
S04	M	Visual	Uses mental imagery
S05	F	Verbal	Reflective, narrative style
S06	F	Symbolic	Good at algebraic manipulation
S07	M	Verbal	Explains concepts narratively

These profiles provided a useful basis for interpreting how each student engaged with the idea of convergence and how their experiences differed depending on their representational habits.

Dominant Mathematical Representation Profiles

Analysis of the students' work revealed tendencies that were consistent with their DMR classification. These tendencies functioned not only as preferences but also as starting points that shaped their interpretations.

Table 2. Overview of Symbolic, Visual, and Verbal Profiles

DMR Type	Example Participants	Strengths	Weaknesses
Symbolic	S01, S02, S06	Procedural accuracy and fluency with notation	Limited intuitive grasp of sequence behavior
Visual	S03, S04	Strong intuitive sense of how sequences behave	Difficulty translating intuition into formal expression
Verbal	S05, S07	Ability to articulate conceptual meaning	Limited symbolic precision

The symbolic group tended to rely on procedural steps, the visual group relied on mental or drawn images to make sense of convergence, and the verbal group used narrative explanations to form meaning. Each approach shed light on different aspects of the learning process.

Cognitive–Affective–Representational Themes

The meaning units generated from interviews, written solutions, and observations were synthesized into broader themes that captured how students interpreted convergence at cognitive, emotional, and representational levels.

Table 3. Cognitive–Affective–Representational Themes

Dimension	Key Findings	Variations Across DMR
Cognitive	Students interpret convergence through different entry points	Symbolic = formal-first; Visual = image-first; Verbal = meaning-first
Affective	Emotional comfort shapes representational choices	Symbolic = stable; Visual = fluctuating; Verbal = hesitant
Transformational	Movement across representations is minimal	Shifts occur tentatively and incompletely

These dimensions reveal that students' engagement with convergent sequences is never purely procedural or conceptual; it is interwoven with confidence, uncertainty, and familiarity with particular modes of thinking.

Consolidated Meaning-Unit Themes

The original meaning-unit tables contained extensive experiential fragments that were consolidated here for clarity while maintaining their conceptual depth.

Table 4. Consolidated Meaning-Unit Themes

Theme	Meaning Units (Condensed)	Evidence Across Participants
Reliance on Familiar Representation	Students begin with the representational mode they trust	S01, S02, S03, S05
Tension During Representation Shift	Attempts to change representation often create confusion	S03, S04, S07
Fragmented Understanding of ε–N	Students recall definition but cannot connect it with behavior	Strong among symbolic students
Visualization as Cognitive Anchor	Diagrams or mental images help form early meaning	S03, S04
Narrative Reasoning for Meaning Making	Verbal descriptions assist conceptual clarification	S05, S07

These themes point to the complex way in which students negotiate between what they know, what they feel confident about, and what the task demands.

Patterns of Representational Transformation

Shifts from one representation to another occurred only occasionally and often with noticeable hesitation. Many students reverted to their dominant representational mode when they felt uncertain.

Table 5. Patterns of Representational Shifts

Participant	Dominant Mode	Attempted Shift	Outcome
S01	Symbolic	None	Remained in symbolic reasoning
S03	Visual	Visual → Symbolic	Attempt was partial and uncertain
S05	Verbal	Verbal → Symbolic	Tried but lacked confidence
S07	Verbal	Verbal → Visual	Helped build initial intuition

These findings suggest that representational flexibility is not naturally internalized and may require structured instructional support.

Discussion

Students' experiences in this study show that understanding convergent sequences involves navigating multiple layers of meaning that extend beyond procedural recall. Several participants approached the tasks with confidence rooted in familiar representations, yet this confidence often narrowed the scope of their reasoning. This finding echoes the nuanced behavior described by Türkmen (2025), who demonstrates that convergence behaves subtly even in advanced fuzzy-paranormed contexts. What emerged here is a similar complexity at a more foundational level, where students struggled to align intuition, symbol, and narrative. Their reflections reveal that grasping a limit requires a delicate balance between formal reasoning and conceptual grounding. When one representational mode dominated too strongly, students overlooked features of convergence that required alternative perspectives. This imbalance created conceptual blind spots that were not immediately visible in their written work. The pattern underscores how deeply representational comfort shapes students' mathematical thinking.

Symbolic-dominant students consistently relied on notation and algebraic procedures to guide their understanding of convergence. Although their solutions appeared structured, their explanations often lacked the conceptual depth needed to justify why a sequence converged. This distinction aligns with observations by Ibrahim (2025), who notes that symbolic form alone cannot guarantee meaningful interpretation in summability theory. Many students in this group reported that the ε–N definition was easy to memorize but difficult to internalize in practice. Their tendency to treat symbolic expressions as self-explanatory hindered their ability to articulate underlying ideas. This separation between manipulation and meaning suggests a gap in how symbolic knowledge is framed during instruction. Without intentional emphasis on conceptual interpretation, symbolic dominance can reinforce superficial understanding. Their narratives highlight the importance of bridging symbolic fluency with intuitive insight.

Students who preferred visual reasoning followed a different path toward understanding convergence. They often relied on mental images or sketches to sense how terms moved toward a limit, yet translating these images into rigorous symbolic language proved challenging. Their difficulty resonates with Çolak (2025), who emphasizes the structural role of visualization in sequence behavior but also notes its limitations when formal proof is required. These students described moments in which their diagrams carried meaning that they felt unable to express mathematically. As they attempted to formalize their thinking, their initial clarity sometimes dissolved into uncertainty. This tension illustrates how intuition can open conceptual doors but may not supply the structure needed for formal justification. Their experiences show that visualization is powerful but incomplete without representational integration. Instruction that explicitly links visual and symbolic forms may help students navigate this transition more smoothly.

Verbal-dominant students made sense of convergence through narrative descriptions, relying on everyday language to articulate how sequences behaved. Their reflections parallel themes identified by Baleanu (2025), who highlights the interpretive weight of descriptive reasoning in general sequence-space theory. These students often expressed convergence clearly in words yet faltered when required to convert those descriptions into symbolic form. They perceived symbolic notation as restrictive, as if it forced them to abandon meaning for structure. This shift created a sense of cognitive dissonance that limited their willingness to engage with formal definitions. Their accounts illustrate how language can function as a productive entry point but may become a barrier when meaning must be distilled into symbols. This dynamic suggests that instruction should emphasize transitions between narrative and formal representations. Without such support, students may struggle to reconcile intuitive explanations with the precision required in analysis.

Emotional factors emerged as a subtle but influential element shaping how students chose and used representations. Symbolic-dominant participants reported feeling secure when manipulating notation but uneasy when asked to rely on intuition. Visual-dominant students expressed fluctuating confidence depending on how clearly they could imagine the sequence's behavior. Verbal-dominant students voiced hesitation when required to formalize their reasoning. These patterns mirror the interplay of affect and cognition highlighted by Yousif (2025) in work on student engagement with abstract structures. Emotional comfort often guided students back to their preferred mode even when it limited their understanding. This tendency suggests that representational choices are not purely cognitive but intertwined with self-perception and confidence. Recognizing this emotional dimension is essential for designing instruction that supports representational flexibility.

Attempts to shift across representations revealed additional challenges faced by students. Some visual-dominant participants tried to express their intuitive reasoning symbolically but struggled to maintain clarity during the transition. Others began with narrative explanations but found the shift toward formal notation cognitively demanding. These experiences resemble the interpretive strain emphasized by Mohammed (2025) in work on sequence-space transformations involving fuzzy structures. In this study, such shifts required both cognitive reorganization and emotional resilience, and many students felt unprepared for either. As confusion increased, they often retreated to their dominant representation, reinforcing rigid patterns rather than challenging them. This retreat shows how representational habits can serve as both anchors and constraints in mathematical reasoning. Their difficulty suggests that representational shifting should be modeled, scaffolded, and practiced explicitly.

Thematic analysis revealed that students' meanings for convergence often remained fragmented despite their representational preferences. Some could restate definitions but could not relate them to the sequential behavior they observed. Others intuitively grasped the idea of approaching a limit but faltered when required to justify that intuition formally. These patterns correspond to the layered interpretive processes documented by Dorai (2018) in approximation theory, where multiple forms of reasoning must be coordinated. Students in this study rarely achieved such coordination, instead relying on a single mode that felt familiar. As a result, their understanding lacked resilience when tasks demanded representational shifts. This fragility illustrates how deeply representational isolation affects conceptual development. Without explicit integration of symbolic, visual, and verbal approaches, students' understanding remains compartmentalized.

The findings also highlight the need for representational coherence when students confront abstract material early in their mathematical development. Visual-dominant students needed more symbolic grounding, symbolic-dominant students needed stronger intuitive anchors, and verbal-dominant students needed bridges connecting their explanations to formal notation. These tensions echo insights from Chil (2025) and Wójtowicz (2025), who emphasize that convergence often depends on weaving together heterogeneous modes of reasoning. In this study, students' preferred modes illuminated some aspects of convergence while obscuring others. They rarely recognized the interpretive trade-offs embedded in their representational choices. Their narratives suggest that coherence across representations is not an innate skill but a relationship that develops through guided practice. Supporting this development may be essential for building robust and transferable understanding.

Students repeatedly pointed to instructional patterns as a major influence on their representational habits. Many recalled lessons that centered almost exclusively on symbolic procedures with minimal attention to intuitive or descriptive reasoning. This imbalance resembles concerns expressed by Yang (2025), who warns that instructional misalignment can hinder students' ability to manage representational complexity in higher mathematics. In this study, lack of explicit representational integration led students to rely heavily on whichever mode felt safest. Over time, their representational habits solidified into rigid patterns that limited adaptability. The absence of instruction encouraging movement across modes further intensified this rigidity. Their accounts demonstrate the importance of designing learning environments that intentionally connect symbolic, visual, and verbal approaches.

Taken together, these findings portray learning about convergent sequences as a multifaceted process shaped by intuition, formal structure, narrative meaning, and affective experience. Representational tendencies acted simultaneously as strengths that supported initial insight and as constraints that limited deeper understanding. The works of Türkmen (2025), Ibrahim (2025), Çolak (2025), Yang (2025), Liu (2025), Dorai (2025), Chil (2025), Wójtowicz (2025), Baleanu (2025), and Yousif (2025) collectively illustrate that convergence requires coordination of multiple interpretive frames. Students in this study encountered those frames not only through formal work but also through the negotiation of meaning within their preferred representational modes. Their reflections show how representational habits become intertwined with mathematical identity over time. At the same time, the findings suggest that representational flexibility must be deliberately cultivated rather than assumed to emerge naturally. Ultimately, these insights point toward instructional practices that emphasize integration across symbolic, visual, and verbal representations to support deeper and more coherent understanding of convergence.

Implications

The results of this study point to several meaningful consequences for the teaching and learning of real analysis, especially when students encounter ideas such as convergent sequences for the first time. The participants' experiences show that the ways they choose to represent mathematical ideas shape not only how they approach a task but also how deeply they understand the underlying concepts. When instruction focuses too narrowly on symbolic procedures, students who rely on visual or verbal reasoning may develop intuitive insight without the structural clarity needed to justify their thinking. Conversely, symbol-oriented students may become proficient at manipulating notation while remaining unsure about the behavior of the sequences they are trying to describe. These patterns indicate that classrooms should encourage students to move more freely between different representational forms rather than treating one mode as the default. They also suggest that emotional comfort—often invisible to instructors—plays a larger role in representational choice than is usually acknowledged. If representation is understood not merely as a tool but as a lens through which students organize their thinking, then fostering representational flexibility becomes an essential part of helping them understand convergence in a meaningful way.

Limitations and Suggestions

Several constraints of the study warrant careful consideration when interpreting the findings. The small number of participants, while appropriate for a phenomenological investigation, limits the extent to which the results can be generalized beyond the immediate context. The students also came from the same academic environment, which may have shaped their representational tendencies in ways that differ from learners in other institutions or curricula. Because much of the data relied on students' own descriptions of their thought processes, some aspects of their reasoning may remain unspoken, simplified, or unintentionally filtered. The tasks used in the study focused specifically on convergent sequences, and it is possible that students might display different representational patterns when working with other topics in mathematics. Furthermore, prior instructional experiences could not be isolated completely, making it difficult to determine how much of their representational behavior resulted from personal preference and how much from exposure to particular teaching styles. Although the combination of interviews and written work offered rich detail, the depth of insight depended on how willing and able each student was to reflect on their own thinking. These limitations suggest that the findings should be seen as context-dependent insights rather than universal claims.

The study opens several possibilities for future research and instructional refinement. Expanding the investigation to include a larger and more diverse group of students could reveal whether certain representational habits are widespread or specific to particular learning environments. It may also be useful to explore targeted classroom interventions designed to help students translate ideas across symbolic, visual, and verbal modes, especially when dealing with abstract material such as limits. Researchers could examine whether representational flexibility strengthens over time when students engage in structured reflection on why they choose certain representations and how those choices influence their understanding. Teachers might consider introducing activities that ask students to compare different representations of the same mathematical idea, giving them deliberate practice in shifting between perspectives. Curriculum developers could design materials that integrate visual and narrative reasoning more closely with formal notation, reducing the gap that many students experience when moving into symbolic territory. Another promising direction involves studying how students' emotions influence their representational decisions and how supportive learning climates can help mitigate representational anxiety. Taken together, these suggestions highlight the importance of treating representation as a dynamic process rather than a fixed skill, one that evolves through guidance, practice, and thoughtful instructional design.

CONCLUSION

This study reveals that students make sense of convergent sequences through representational habits that shape how ideas are noticed, interpreted, and justified, and these habits often guide their thinking more strongly than the formal definitions they have learned. The participants' reflections show that symbolic fluency, visual intuition, and verbal explanation each provide meaningful entry points into the concept, yet none of these modes alone is sufficient for developing a complete understanding. When students relied too heavily on one form of representation, their reasoning tended to narrow, leaving important aspects of convergence unexplored or only partially understood. Their difficulties in moving between representations also indicate that representational flexibility is not an automatic outcome of learning but a skill that grows when instruction intentionally encourages students to connect intuition with structure and language with formalism. The patterns observed here suggest that teaching convergence effectively requires more than emphasizing procedures or definitions; it calls for learning environments that help students weave together multiple ways of thinking so that their understanding becomes more coherent and resilient. By acknowledging the interpretive, emotional, and symbolic dimensions of students' experiences, educators can create opportunities for learners to build richer and more integrated insights into real analysis.

AUTHOR CONTRIBUTIONS STATEMENT

NU initiated the research idea, coordinated the overall study design, and led the data collection process, including classroom observations, documentation analysis, and interviews.
SR contributed to developing the methodological framework, refined the structure of the phenomenological procedures, and supported the validation of emerging themes during data analysis.
PA assisted in instrument development, helped verify the consistency of the findings, and provided input that strengthened the alignment between data sources and the study's conceptual framework.
SU played a key role in interpreting the results, offering critical perspectives during the drafting of the results and discussion sections, and reviewing the coherence between the different stages of analysis.
IW oversaw the research process as a whole, advised on theoretical grounding, ensured methodological rigor, and contributed to the final revision of the manuscript to enhance clarity and scholarly quality. All authors reviewed, contributed to, and approved the final version of this manuscript.

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