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Praxology in advanced geometry textbooks for distance education: A hermeneutic review of structure and knowledge representation

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Abstract

Background: Geometry plays a fundamental role in mathematics education by developing logical reasoning and spatial understanding. Despite its importance, geometry remains a difficult subject for university students, particularly in distance learning contexts. While several studies have analyzed geometry textbooks, few have examined their knowledge structures through a praxeological perspective.
Aim: This study aims to analyze a university-level Euclidean geometry textbook by identifying how the components of praxeology, namely task (T), technique (τ), technology (θ), and theory (Θ), are organized and interconnected to support meaningful learning.
Method: The research applied a hermeneutic phenomenological design. The textbook, used in a master's geometry course at an Indonesian university, was analyzed through repeated readings and qualitative interpretation. Data were coded and categorized according to the praxeological framework and validated through researcher discussions.
Result: The findings show that the textbook demonstrates a coherent praxeological structure with accurate theoretical explanations and effective technological representations. However, the analysis revealed weaknesses such as limited rationale for applying specific techniques, insufficient connection between theoretical concepts and exercises, and few examples of proofs.
Conclusion: The study concludes that while the textbook reflects strong praxeological principles, improvements are needed in clarifying technique rationales, linking theory and practice, and structuring technological components. The results provide pedagogical insights for developing university geometry textbooks that enhance conceptual understanding, reflective reasoning, and learning effectiveness in both traditional and distance education settings.

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INTRODUCTION

Euclidean geometry is a fundamental domain of mathematics that plays a central role in developing students' logical reasoning and spatial understanding (BSKAP, 2022; Isnawan, 2023; Weigand et al., 2025; Sudirman et al., 2021; 2022; 2023). Similar to mathematics in general, which serves various intellectual and practical purposes (Hadi et al., 2025; Teh et al., 2025; Zu & Yow, 2024), geometry contributes not only to the advancement of mathematical thinking but also to a wide range of scientific and everyday applications (Burlacu & Mihai, 2023; Sudirman et al., 2023; Sun, 2024; Yumiati et al., 2024). In architecture, for example, geometry supports the design and construction of

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buildings by ensuring structural balance, precision, and visual harmony (Basri et al., 2024; Miao et al., 2022; Sudirman et al., 2025; Takva et al., 2023). In the field of aerospace, geometric reasoning is equally indispensable, helping scientists plan satellite trajectories, design rockets, and analyze spatial configurations in orbital mechanics (Ma et al., 2019; Zardashti & Emami, 2021). Considering its broad relevance across disciplines and real-world problem solving, geometry education at the tertiary level should be designed not only to build conceptual mastery but also to strengthen students' ability to apply geometric reasoning in both academic and professional contexts. Yet these strengths do not automatically translate into positive learning experiences for all university students.

Despite these broad contributions, empirical evidence shows that geometry is often less favored by university students and is associated with suboptimal learning outcomes (Ali et al., 2024; Doz et al., 2022; Uzun & Özturk, 2023; Yorulmaz & Çilingir Altıner, 2021), a pattern that is even more visible in online learning contexts (Csiba & Vajo, 2024; Listiani & Saragih, 2022). Several factors contribute to this situation, including the abstract character of geometric concepts, limited spatial reasoning skills among students, and lecture-centered approaches that provide few opportunities to build deep conceptual understanding. These challenges tend to intensify in distance education, where learners must work more independently and frequently lack hands-on exploration or guided interaction, which are critical for making sense of spatial relationships. As a result, many students struggle with foundational principles and carry these difficulties into broader mathematical performance. Taken together, these issues signal an urgent need to rethink how geometry is taught at the tertiary level, with particular attention to technology-supported, active learning designs and well-structured instructional resources that make concepts more accessible.

Multiple factors have been linked to suboptimal learning outcomes in geometry courses, and one salient factor is the quality of the textbooks that students use (Fitriyani et al., 2023; Greenberg, 2010; Novita et al., 2018; Toybah et al., 2020; Weigand et al., 2025). Textbooks strongly shape what and how students learn, and their impact on achievement has been documented in prior syntheses and empirical studies (Jang et al., 2016; Wijaya et al., 2022). Materials that are comprehensive, accessible, and engaging tend to cultivate interest and motivation, which in turn support higher performance in geometry courses (Li & Wang, 2024; Wijaya et al., 2022). Despite these indications, relatively few studies have examined textbooks specifically designed for university-level geometry, and even fewer have analyzed them through a praxeological perspective that foregrounds the meaning-making processes embedded in mathematical practice. Taken together, these observations point to a clear gap that warrants a focused examination of how advanced geometry textbooks structure tasks, techniques, technologies, and theories for effective learning.

Prior research has illuminated patterns in school-level geometry textbooks yet has not sufficiently addressed university materials from a praxeological standpoint. Yunianta et al. (2023) analyzed the praxeological structure of three-dimensional geometry textbooks used in Indonesian secondary schools using a didactical praxeology framework and found imbalances in the depth of the four components, namely task (T), technique (τ), technology (θ), and theory (Θ). While informative about school contexts, this work did not examine textbooks at the university level where epistemic demands and representational complexity are typically higher. In a related study, Wang and Yang (2016) conducted a comparative content analysis of elementary geometry textbooks across five countries, namely Finland, China, Singapore, Taiwan, and the United States, and reported significant differences in representations, question types, and task formats. Although relevant to the general framing of geometry in basic education, this study also focused on the school level rather than higher education. Collectively, these findings underscore a gap concerning how praxeological components are organized and justified in university geometry textbooks, which motivates the present investigation.

To situate the present study within the literature, it is useful to clarify how earlier textbook analyses differ from our focus. Toybah et al. (2020) conducted a descriptive needs analysis to identify the requirements for a scientific approach-based geometry and measurement textbook for prospective primary school teachers, and their study indicated a lack of adequate scientific textbooks while recommending the development of such resources. Building on this, a comparative overview clarifies how previous studies differ from the present work: Yunianta et al. (2023) examined secondary school textbooks using a didactical praxeology lens, Wang and Yang (2016) compared primary school textbooks through content analysis across five countries, and Toybah et al. (2020) focused on undergraduate courses for pre-service primary teachers using descriptive needs analysis, whereas the current research analyzes a graduate-level textbook in mathematics education using a praxeological approach. These studies illuminate school-level materials and needs but do not address how praxeological components are organized in advanced, university-level resources. Against this backdrop, the present research examines a graduate-level geometry textbook through a praxeological lens to foreground structures that support conceptual understanding in higher education.

Based on the aforementioned background, the present study aims to examine a university-level Euclidean geometry textbook using a didactical praxeology perspective. The findings are expected to serve as a foundation for developing a scientific-approach-based geometry textbook that can be used in university geometry courses. The focus of this study is on Euclidean geometry, as it is a central and widely taught topic in university geometry courses (Marange & Tatira, 2023; Wijayanti et al., 2021). To address the research objective, the following research questions are formulated:

1) How is the task (T) component of Euclidean geometry represented in the textbook?
2) How are the techniques (τ) for learning Euclidean geometry described in the textbook?
3) How is technology (θ) integrated into the learning of Euclidean geometry in the textbook?
4) How is the theoretical (Θ) component presented to support the understanding of Euclidean geometry?

METHOD

Research Design

This study employs a hermeneutic phenomenological design. This choice is based on the study's objective to analyze the meaning of an individual's experience (the textbook author) within a specific context (Euclidean Geometry) through interpretation (hermeneutics) and phenomenological awareness (direct experience) as reflected in the textbook (Fiantika et al., 2018; Fuster Guillen, 2019; Isnawan et al., 2023; Kafle, 2011; Roubach, 2023). In simpler terms, the hermeneutic phenomenology in this study is applied by interpreting the textbook author's understanding of Euclidean Geometry as presented in the textbook. The textbook analyzed in this study is a core instructional material for a geometry course used at a public university in Indonesia. It was selected because it serves as the primary reference for learning geometry in that institution.

The hermeneutic phenomenological procedure applied in this research involves several steps (Fuster Guillen, 2019; Isnawan et al., 2022). First, the researchers determined the focus of the study. In this context, geometry was chosen as the focus because it constitutes a fundamental content element in mathematics. Second, the researchers selected a textbook. While many geometry textbooks are used in universities, not all are suitable for master's programs. Therefore, this study utilizes a geometry textbook used in a mathematics education master's program due to its advanced content. Third, the researchers selected a specific topic for analysis, namely Euclidean Geometry.

Fourth, the researchers repeatedly read the selected topic in the textbook and sought to understand it in depth. Fifth, the narrative presented by the author was analyzed using the praxeological framework (Kafle, 2011; Roubach, 2023). The framework applied consists of task (T), technique (τ), technology (θ), and theory (Θ) (Hendriyanto et al., 2023; Pocalana & Robutti, 2024). Sixth, the researchers assessed the textbook's strengths and proposed recommendations for improvement. An overview of this research procedure is presented in Figure 1.

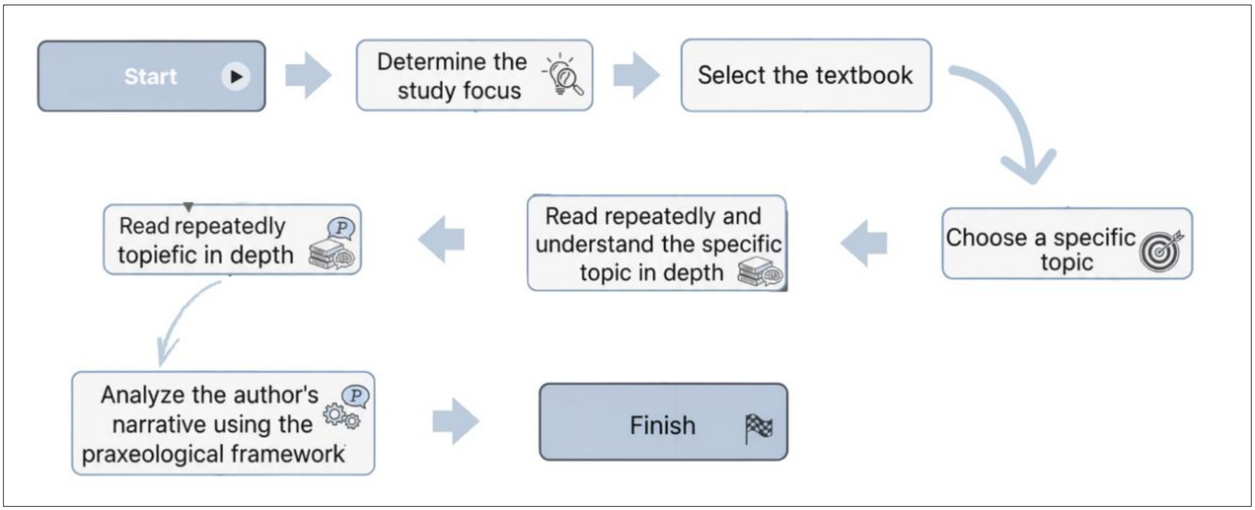


Figure 1. Research Procedure

Textbook Analyzed

As previously described, the analyzed textbook is used in a geometry course at a public university in Indonesia. Although various textbooks are available, this particular one is used by graduate students in mathematics education. It was chosen because it covers both fundamental and advanced geometry topics. Additionally, its advanced level allows for various theoretical and technological interpretations by the authors. The textbook also serves as the main instructional resource for students studying geometry at the graduate level.

Moreover, the textbook is published by the university itself, ensuring standardized writing and material presentation. It is authored by a team rather than a single author, bringing diverse perspectives on geometry topics. The textbook's writing process, peer-review, revisions, and layout are all meticulously managed, which adds to its appeal for this research. Finally, it is commonly used in distance learning contexts (online and hybrid), giving it structural features distinct from conventional textbooks.

Data Collection Procedures

Data collection in this study was conducted through repeated readings of the textbook. In addition to reading, the researchers aimed to comprehend each concept presented in the text. The researchers also solved practice problems provided in the textbook to gain deeper insight into the author's intentions as conveyed in the narrative. Supplementary geometry textbooks and other references were consulted to enhance the researchers' understanding, particularly of Euclidean Geometry. Once the researchers felt confident in their understanding of the narrative, they analyzed and identified the components of the praxeological framework: task (T), technique (τ), technology (θ), and theory (Θ). The researchers then used the primary textbook and other references to evaluate the textbook's strengths and identify areas for improvement.

Data Analysis Methods

The data analysis method employed in this study is qualitative analysis aimed at uncovering the meaning behind the narratives presented by the textbook authors (Isnawan et al., 2024; Miles et al., 2014). The qualitative analysis was conducted alongside other research activities such as data collection and writing of the findings. The analysis followed three key stages. First, the researchers selected specific topics from the textbook and organized them into a praxeological table comprising task (T), technique (τ), technology (θ), and theory (Θ). This task was performed by all researchers to capture multiple perspectives.

In the second stage, each researcher assigned codes or evaluations to the descriptions of each component in the praxeological table. In the final stage, the researchers compared their coding results in a focus group discussion (FGD) to review and confirm whether the codes adhered to the principles of Euclidean Geometry. In cases of discrepancy, efforts were made to reach consensus and refine the coding or descriptions within the praxeological table. These three stages were repeated iteratively to ensure accuracy and completeness in identifying the components of task (T), technique (τ), technology (θ), and theory (Θ) during the analysis process.

RESULTS AND DISCUSSION

Results

Context of Euclidean Geometry

Euclidean geometry is a branch of geometry that studies shapes and space based on Euclid's axioms; in two dimensions, it can be represented on the coordinate plane R^2 (Gröger, 2021). Euclidean geometry enables the representation of various geometric objects, such as points, lines, and vectors, using numbers and equations (Bahreyni et al., 2024). It is founded upon the concepts of vector length, vector addition and scalar multiplication (vector operations), inner products of vectors, and the concept of distance between points with specific properties (Heijungs, 2025). Euclidean geometry allows for the systematic analysis of spatial relationships and geometric properties through various algebraic calculations (Korchmaros, 2025).

Euclidean geometry offers numerous benefits, both within the context of mathematics itself and in everyday life (Marange & Tatira, 2023). In mathematics, Euclidean geometry serves as a foundation for studying analytic geometry, as it aids in the representation of geometric objects using an algebraic approach (Doré & Broda, 2019). It also assists learners in interpreting geometrical forms through derivatives or integrals in calculus, provides concrete visualizations of abstract concepts in linear algebra, and forms the basis for exploring differential geometry and trigonometry (Bašić & Milin Šipuš, 2022).

Moreover, Euclidean geometry contributes to the advancement of other scientific disciplines (Mlambo & Sotsaka, 2025). For instance, it plays a crucial role in physics by helping to determine position, velocity, and acceleration in classical mechanics (Elshenhab et al., 2022). It is also useful in describing field configurations in electromagnetic theory (Giglio & Rodrigues, 2012). Beyond science, Euclidean geometry aids in instilling essential principles in architecture and construction, particularly in designing structurally stable buildings, supporting graphic design from various perspectives, and serving as the foundational system for location determination in navigation and mapping (Liapi, 2002).

Praxeological Analysis Results of the Euclidean Geometry Context

1. How is the Task (T) Description of Euclidean Geometry Presented in the Textbook?

In this study, the term *Task (T)* refers to a variety of problems or questions that can be addressed using the Euclidean Geometry content taught. The description of Task (T), derived from thematic analysis, is presented in Table 1. An example of Task 1 (T-1) is shown in Figure 2.

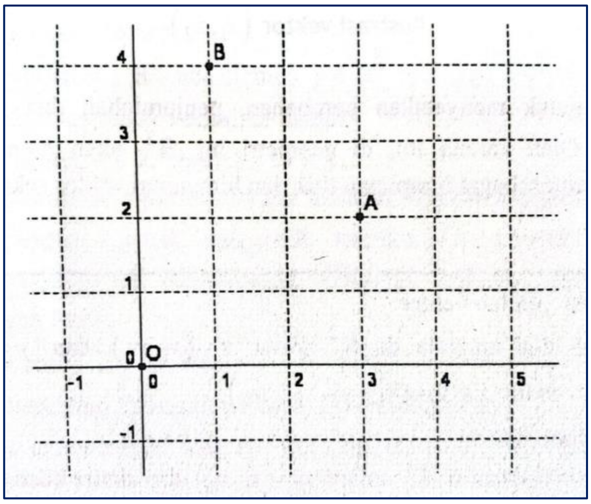


Figure 2. T-1 Trailer

Table 2. Results of the praxeological analysis of the task (T)

Code	Description
T-1	Identify and draw points and vectors on the coordinate plane.
T-2	Perform basic vector operations (vector addition and scalar multiplication).
T-3	Determine the length of a vector.
T-4	Calculate the distance between two points on the Euclidean plane.
T-5	Write the equation of a line on the Euclidean plane in various forms.
T-6	Determine the conditions for two lines to be parallel or perpendicular.
T-7	Determine the point of intersection of two lines.
T-8	Analyze geometric relationships using vectors (orthogonal projection).

The Table 2 presents a variety of tasks (Task/T) in Euclidean Geometry learning, identified through praxeological analysis, reflecting a progressive structure from basic to advanced concepts. Each task is designed to develop students' competencies in understanding and applying vector and analytic geometry concepts, starting from identifying points and vectors on the coordinate plane (T-1), performing basic vector operations (T-2), to more complex analytical skills such as writing equations of lines (T-5), determining relationships between lines (T-6), and analyzing geometric relationships using orthogonal projection (T-8). Overall, these tasks represent a systematic approach to fostering both conceptual understanding and procedural fluency in coordinate- and vector-based Euclidean Geometry.

How is the Technique (τ) Description of Euclidean Geometry Presented in the Textbook?

The technique (τ) refers to the procedures or methods employed to solve the tasks (T). The description of the technique (τ) can be found in Table 2. Each technique (τ) in Table 2 corresponds sequentially and individually to the tasks (T) presented in Table 1.

Table 2. Results of the praxeological analysis of techniques (τ)

Code	**Description**
τ-1	Placing the point (x_1, x_2) on the Cartesian plane with a horizontal axis (x_1) and a vertical axis (x_2). Drawing a vector as a directed line segment from the origin (0,0) to the point (x_1, x_2).
τ-2	Vector addition is performed by adding corresponding components: $(x_1 + y_1, x_2 + y_2)$. Scalar multiplication involves multiplying each component of the vector by a scalar α: $(\alpha x_1, \alpha x_2)$.
τ-3	Using the Pythagorean theorem in coordinate form to calculate the square root of the sum of the squares of the vector components.
τ-4	Applying the Euclidean distance formula by computing the squared differences of the x- and y-coordinates, summing them, and then taking the square root. Alternatively, the “city block” or Manhattan distance can be calculated by summing the absolute values of the differences of the coordinates.
τ-5	The form $ax + by = c$ is used to identify two points on the line to determine the direction vector or normal vector. The vector form $X = P + tv$ is used to select a point P on the line and determine the direction vector v (e.g., from the difference of two points). The normal form $\langle X - P, N \rangle = 0$ is used by identifying a point P on the line and a normal vector N. Algebraic manipulation is performed to convert from one form to another.

τ-6	Parallel lines: For *ax + by = c*, compare the ratios of the coefficients ($a_1/a_2 = b_1/b_2$); for vector/normal forms, the direction or normal vectors must be scalar multiples of each other. Perpendicular lines: For *ax + by = c*, the dot product of the normal vectors must be zero ($a_1a_2 + b_1b_2 = 0$); for vector/normal forms, the dot product of the direction or normal vectors must also be zero.
τ-7	To find the intersection point of two lines, equate the two vector-parametric equations, form a system of linear equations, solve for the parameters *t* or *s*, and substitute back to obtain the coordinates of the intersection point.
τ-8	Use the concept of vector projection: $x = \frac{\langle x,u\rangle}{\|u\|^2}u + \frac{\langle x,v\rangle}{\|v\|^2}v$ (if u and v are orthogonal), or use the general formula for vector projection.

This Table 2 represents the technical component (τ) within the framework of didactical praxeology, illustrating how each task (T) in Euclidean Geometry learning is addressed through specific and systematic mathematical procedures. These techniques not only describe algorithmic steps, such as drawing vectors, adding components, or applying the Euclidean distance formula, but also reflect the kind of *mathematical know-how* assumed by the textbook as essential student competencies. Furthermore, the presence of multiple forms of representation (algebraic, geometric, and vectorial) within these techniques indicates that the textbook encourages transitions between representations and promotes a flexible understanding of geometric objects. The techniques for determining line relationships (parallelism and perpendicularity), points of intersection, and orthogonal projections require a deep comprehension of spatial structures and fundamental properties within Euclidean space. Thus, the table goes beyond listing procedures; it also reveals an underlying epistemological approach to how geometric knowledge is framed in the textbook, emphasizing formal rationality, logical coherence, and mathematical generalization as central to the practice of Euclidean geometry in formal education contexts.

How is the Description of Technology (θ) in Euclidean Geometry Presented in the Textbook?

In the context of this praxeological study, technology (θ) refers to the tools and representations used to support the implementation of techniques. A complete description is provided in Table 3.

Table 3. Results of the praxeological analysis of technology (θ)

Code	Description
θ-1	Cartesian coordinate plane. A graph with x_1 and x_2 axes (or x and y), equipped with a grid scale (dashed lines) to visualize the positions of points and vectors.
θ-2	Vector notation. Representation of points and vectors as ordered pairs (x_1, x_2) or (x, y).
θ-3	Vector operation notation. Use of the symbol + for vector addition and scalar multiplication (e.g., αx).
θ-4	Notation for distance and vector magnitude. Use of the symbol $\|x\|$ for the magnitude of a vector and d(P, Q) for the distance between two points.
θ-5	Various forms of line equations. Algebraic notation (ax + by = c), vector-parametric form (X = P + tv), and normal form (⟨X − P, N⟩ = 0).
θ-6	Common mathematical symbols, such as ∈ (element of), ℝ (set of real numbers), √ (square root), ⟨·,·⟩ (inner product), and t ∈ ℝ (parameter).

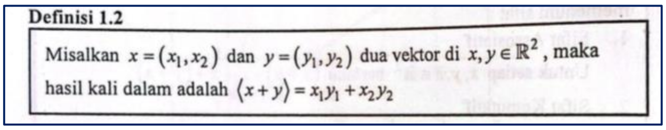
This Table 3 presents the technological component (θ) within the framework of didactical praxeology, functioning as an epistemological bridge between techniques (τ) and theoretical justifications (Θ) in the learning of Euclidean Geometry. The listed technological elements, such as the Cartesian coordinate system, vector and operation notations, mathematical symbols, and various forms of line equations, serve not merely as visual or symbolic aids, but as conceptual structures through which mathematical objects are represented and understood. In this context, technology is not simply a procedural tool but an integral part of the mathematical way of thinking shaped by representational systems. For example, notations like ⟨X − P, N⟩ = 0 or $\|x\|$ do more than simplify mathematical expressions, they condition how students conceptualize spatial relations such as orthogonality, distance, and direction. Thus, the technologies described in the table reflect how the textbook constructs geometric reality through representational systems that enable abstraction, generalization, and coordination across graphical, symbolic, and algebraic modes. Technology (θ) here underscores that learning Euclidean Geometry is not merely about "drawing" or "calculating," but about mastering the formal language and representational structures that allow students to think mathematically within the Euclidean spatial framework.

How is the Theoretical (Θ) Description of Euclidean Geometry Presented in the Textbook?

In this praxeological study, theory (Θ) refers to the mathematical principles or justifications that explain why a given technique (τ) works. A complete description of the theoretical components can be found in Table 4. An excerpt of Θ-3 is illustrated in Figure 3.

Table 4. Results of the Praxeological Analysis of Theory (Θ)

Code	Description
Θ-1	The concept of the vector space $\mathbb{R}^2$. $\mathbb{R}^2$ is defined as the set of all ordered pairs of real numbers, equipped with vector addition and scalar multiplication. Elements of $\mathbb{R}^2$ may be interpreted either as points or as vectors.
Θ-2	Properties of vector operations. Theorem 1.1 outlines the properties of a vector space: associativity, commutativity, existence of an identity element (the zero vector O(0,0)), existence of an inverse element, scalar identity (1x = x), distributivity of scalar multiplication over vector addition, distributivity of scalar multiplication over scalar addition, and scalar multiplication associativity. These axioms form the foundational structure that qualifies $\mathbb{R}^2$ as a vector space.
Θ-3	The concept of the dot product. The dot product is defined as ⟨x, y⟩ = $x_1y_1 + x_2y_2$. Theorem 1.2 presents properties of the dot product: additivity, scalar multiplication, symmetry, and the condition ⟨x, x⟩ = 0 if and only if x = O. The concept of orthogonality is introduced: two vectors u and v are orthogonal if ⟨u, v⟩ = 0.
Θ-4	Definition of vector magnitude. Defined in terms of the dot product: $\|x\| = \sqrt{\langle x_1, x_2\rangle} = \sqrt{\langle x_1^2, x_2^2\rangle}$. Theorem 1.3 outlines properties of vector magnitude: $\|x\| \geq 0$, $\|x\| = 0 \Leftrightarrow x = O$, $\|cx\|=\|c\|\|x\|$, and the triangle inequality ($\|x+y\| \leq \|x\| + \|y\|$).
Θ-5	Definition of the distance between two points in Euclidean space. Defined as $d(P,Q) = \|P-Q\| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$. Theorem 1.4 describes the properties of distance: $d(P,Q) \geq 0$, $d(P,Q) = 0 \Leftrightarrow P = Q$, $d(P,Q) = d(Q,P)$, and the triangle inequality ($d(P,Q) + d(Q,R) \geq d(P,R)$). The concept of Euclidean geometry emerges from this distance definition, as it satisfies the fundamental properties of a metric space.
Θ-6	The concept of lines in the Euclidean plane. A line is defined as the set of all vectors that are scalar multiples of a direction vector v (X = tv), or more generally, a line through a point P in the direction of vector v (X = P + tv). The normal line form ⟨X − P, N⟩ = 0 introduces the idea of a normal vector N orthogonal to the line. This component includes the relationships among various forms of line equations.
Θ-7	Properties of parallelism and perpendicularity of lines. Based on the concept of direction or normal vectors: two lines are parallel if their direction/normal vectors are scalar multiples of one another, and perpendicular if their direction/normal vectors are orthogonal (i.e., their dot product is zero).


Definisi 1.2

Misalkan $x = (x_1, x_2)$ dan $y = (y_1, y_2)$ dua vektor di $x, y \in \mathbb{R}^2$, maka hasil kali dalam adalah $\langle x + y\rangle = x_1y_1 + x_2y_2$

Figure 3. Excerpt of Θ-3 (the concept of dot product)

This Table 4 presents the theoretical component (Θ) within the framework of didactical praxeology, serving as the conceptual foundation and mathematical justification for each technique (τ) employed in the teaching of Euclidean Geometry. Unlike techniques, which are procedural in nature, theory provides the "why" behind the "how"**,** that is, the rational basis for why a particular mathematical procedure is valid and applicable. Each entry in the table articulates the axiomatic and theorematic structure of key concepts such as vector spaces, dot product, magnitude, distance, and the properties of lines and their relationships (e.g., parallelism and orthogonality). For instance, the definition and properties of the dot product (Θ-3) not only explain how vectors are multiplied, but also form the basis for the concept of orthogonality, which is central to spatial reasoning. Likewise, the definitions of magnitude and distance (Θ-4 and Θ-5) are not isolated procedures but are intrinsically linked to the axioms of vector spaces and metric properties, thereby constructing a coherent and structured Euclidean framework. This interpretation highlights that theory (Θ) in the textbook is not merely a formal supplement, but a core epistemological component that provides legitimacy, logical coherence, and conceptual depth to the development of geometric knowledge. Therefore, understanding theory in this context not only strengthens students' technical proficiency but also fosters reflective, principled, and formally grounded mathematical thinking.

Discussion

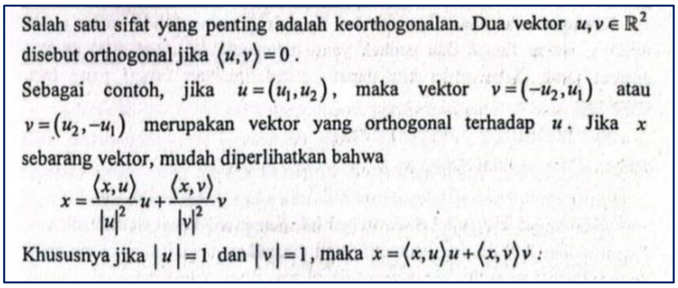
Several recommendations are proposed to enhance the quality of the analyzed textbook. First, there is a lack of explanation regarding why certain techniques (τ) are used to solve specific tasks (T). This observation is consistent with findings by Weinberg & Wiesner (2011), who revealed that many mathematics textbooks do not optimally explain the rationale behind the choice of specific techniques for solving tasks. However, articulating such reasoning helps learners achieve a deeper understanding (Evans et al., 2022). This aligns with research by Vivanco-Galván et al. (2024) and Westley (2024), which emphasizes that justifying the use of specific techniques contributes to more meaningful learning.

Second, there exists a gap between theory (Θ) and practice exercises. The textbook tends to present exercises immediately after theoretical explanations, without offering illustrative examples. This may create learning barriers for students. This finding aligns with Sunday (2014), who observed that some mathematics textbooks provide exercises without prior examples, potentially hindering students' conceptual understanding (Masina & Mosvold, 2023). Similarly, Azzahra & Herman (2022) and Cuarteros & Roble (2024) found that a lack of continuity between theoretical concepts and exercises can create obstacles to learning mathematics effectively.

Another recommendation concerns the structured introduction of technology (θ). Although the textbook employs various forms of notation and graphics, it lacks clear descriptions of when and why specific technological tools (θ) should be used. For instance, it would be beneficial to explain when it is more efficient to use the standard form of a line (ax + by = c) versus its parametric form (X = P + tv). This suggestion is supported by Rustam et al. (2024), who highlight the importance of explaining the rationale behind technological choices to enhance learners' understanding. Similarly, Raave et al. (2024) emphasize that such descriptions help learners decide when to appropriately employ certain technological representations for solving tasks (T).

Furthermore, textbooks should focus more on the categorization of task types (T), which helps students better understand the spectrum of problems they are expected to solve. While the textbook provides various exercises, it would be more helpful if tasks were classified by type, e.g., computational, drawing, proving, or identifying equations. This classification would allow students to grasp the nature of the task more directly. This suggestion aligns with Coppens et al. (2021), who found that task categorization supports learners in narrowing their focus during problem-solving. Mitchell & Carbone (2011) and Scheja & Rott (2024) also affirm that identifying task types helps students tackle problems more efficiently and effectively.

A further area for improvement is the limited integration of orthogonal projection tasks (T-8). Although the textbook discusses the concept of orthogonal projection on pages 1.9 and 1.18, it does not provide a structured, step-by-step explanation of the technique (τ) before introducing related exercises. Figure 4 illustrates a snippet of how orthogonal projection is presented. This gap likely hinders students' ability to transition from theory (Θ) to practice (T). This observation is supported by Chivai et al. (2023), who argue that orthogonal projection is often difficult for students to grasp without clear procedural guidance.


Salah satu sifat yang penting adalah keorthogonalan. Dua vektor $u, v \in \mathbb{R}^2$ disebut orthogonal jika $\langle u, v \rangle = 0$.
Sebagai contoh, jika $u = (u_1, u_2)$, maka vektor $v = (-u_2, u_1)$ atau $v = (u_2, -u_1)$ merupakan vektor yang orthogonal terhadap u. Jika x sebarang vektor, mudah diperlihatkan bahwa

$$x = \frac{\langle x, u \rangle}{|u|^2} u + \frac{\langle x, v \rangle}{|v|^2} v$$

Khususnya jika $|u| = 1$ dan $|v| = 1$, maka $x = \langle x, u \rangle u + \langle x, v \rangle v$;

Figure 4. Snippet of Orthogonal Projection Concept

Another recommendation pertains to proof-related tasks. The textbook frequently delegates the responsibility of constructing proofs to students. While the intended goal may be to foster critical thinking, this approach may pose challenges for students with inadequate prerequisite knowledge. It is advisable that the textbook provides simple proof examples or proof frameworks that students can follow. These examples could serve as additional variations of techniques (τ) in the textbook, especially in online learning environments. This recommendation aligns with Basir & Wijayanti (2020), who found that assigning proof tasks without any guidance often leads to difficulties. Laili & Siswono (2020) also emphasize that students, especially those with limited prior knowledge, require at least scaffolds or cues to succeed in constructing mathematical proofs.

Finally, stronger emphasis should be placed on highlighting conceptual connections within the theoretical components (Θ). For example, the textbook should more explicitly explain the definition of distance (Definition 1.2) in Euclidean Geometry and why it is important. Furthermore, the relationships between successive theorems should be described more coherently. Explaining the rationale behind the sequence of theorems, why a particular theorem follows another, can improve the logical flow of the material. This suggestion is in line with Çakıroğlu et al. (2023), who emphasize that understanding the purpose and application of definitions and theorems leads to deeper comprehension of mathematical concepts. Mayerhofer et al. (2024) also support the view that when students perceive a concept as significant, they are more focused and motivated in their learning.

Implications

The findings of this study imply that a well-structured praxeological organization within geometry textbooks can significantly enhance students' conceptual understanding, particularly in distance-learning contexts. Clear alignment between tasks, techniques, technologies, and theories helps students grasp not only how to solve problems but also why certain procedures are appropriate. This means that textbook authors and instructors should design learning materials that explicitly connect theoretical explanations with procedural steps and provide sufficient examples before independent exercises. Strengthening these links can support deeper reasoning, reduce learning obstacles typically found in abstract geometry topics, and ultimately foster more meaningful engagement with Euclidean concepts

Limitations and Suggestions

This study is limited by its focus on a single graduate-level geometry textbook from one institution, which restricts the generalizability of the findings. Additionally, the analysis is based solely on document review without investigating how students actually interact with the textbook in real learning situations. Future research should incorporate multiple textbooks and classroom-based evidence to evaluate the practical impact of praxeological structures on students' understanding.

Based on these limitations, it is suggested that future textbook development include clearer rationales for technique selection, more structured transitions from theory to practice, and explicit guidance for using different mathematical representations. Instructors are also encouraged to provide additional scaffolding during instruction, especially for complex topics such as orthogonal projection and proofs, to ensure that students can effectively navigate abstract concepts in both face-to-face and distance-learning environments.

CONCLUSION

This study concludes that the analyzed Euclidean geometry textbook demonstrates a generally strong praxeological structure, with clear progression from tasks to techniques, technologies, and theories that support conceptual understanding. Nevertheless, several weaknesses were identified, particularly the limited explanation of why specific techniques are appropriate for certain tasks, the insufficient linkage between theoretical concepts and practice, and the lack of structured examples for complex topics such as orthogonal projection and proof construction. These findings highlight the need for more explicit rationales, improved scaffolding, and better integration of representational technologies to enhance learning effectiveness, especially in distance-education settings. The study underscores the importance of praxeology-informed textbook design and invites future research to refine and evaluate revisions that can strengthen students' reasoning, problem-solving abilities, and overall engagement with advanced geometry.

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AUTHOR CONTRIBUTIONS STATEMENT

All authors contributed significantly to the completion of this research. The specific contributions of each author are as follows:

1. Ma'rufi: Responsible for the conceptualization and design of the study, development of research instruments, data collection, and initial drafting of the manuscript.
2. Muhammad Ilyas: Contributed to data analysis, interpretation of results, and critical revisions to the manuscript to ensure intellectual content and clarity.
3. Nur Wahidin Ashari: Provided expertise in mathematical reasoning and thinking styles, validated the research framework, and contributed to the review of related literature.
4. Tri Bondan Kriswinarso: Assisted in data collection, organization of raw data, and preparation of visual materials, such as tables and figures.
5. Salwah: Supervised the overall research process, reviewed and refined the manuscript, and ensured alignment with journal submission requirements.

All authors have read and approved the final manuscript and agree to be accountable for all aspects of the work.

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