



Development of an Islamic values-based number theory textbook to improve conceptual understanding in mathematics education students

Fatrima Santri Syafri*UIN Fatmawati Sukarno Bengkulu,
INDONESIA**Dodi Isran**STIES NU Bengkulu,
INDONESIA

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Abstract

Background: Number theory is frequently perceived by students as a challenging subject, particularly when instruction relies on traditional textbooks. This issue underscores the necessity for a more engaging, interactive resource that aligns with both learners' preferences and curricular standards.

Aim. This study endeavored to create a comprehensive, curriculum-aligned textbook for teaching number theory to students in the Mathematics Education Program at UIN Fatmawati Sukarno Bengkulu, with the goal of enhancing their conceptual understanding.

Method. We applied the ADDIE (Analysis, Design, Development, Implementation, Evaluation) framework to guide textbook creation. Initially, we conducted a needs assessment involving 49 students, who were then divided into an Experimental Group (using the newly developed textbook) and a Control Group (using a conventional text). Effectiveness was measured via N-Gain scores and independent-sample t-tests.

Results. Students in the Experimental Group exhibited significantly higher N-Gain gains than those in the Control Group, indicating that the interactive, curriculum-aligned design substantially improved their comprehension of number theory. Quantitatively, the Experimental Group's average N-Gain surpassed that of the Control Group by 27.79 points ($p < 0.001$).

Conclusion. The newly developed number theory textbook proved both valid and effective in elevating students' understanding. Its integration of interactive elements and alignment with curriculum standards justifies its recommendation as a primary instructional resource in the Mathematics Education Program at UIN Fatmawati Sukarno Bengkulu.

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INTRODUCTION

Textbooks constitute fundamental instructional resources that significantly influence the effectiveness of teaching and learning in higher education (Isran et al., 2025). A well-organized textbook enables students to pursue learning independently and with greater efficacy (Albedaiwi, 2014). Consequently, creating textbooks that align with curricular requirements and address student needs is essential. According to Aulia & Prahmana (2022), instructional materials should be designed to fulfill learners' educational needs, offer alternative resources, and assist instructors in delivering effective instruction. Moreover, thoughtfully crafted textbooks can promote critical thinking, encourage active engagement, and accommodate diverse learning styles. By integrating contemporary educational tools, such as interactive elements and real-world applications, textbooks not only enhance student engagement but also equip learners with practical problem-solving skills relevant to their future careers.

In academic program development, a study program's unique identity is frequently manifested in its curriculum documents. Higher education plays a pivotal role in nurturing individual intellectual growth, especially in mathematics and its subfields such as number theory. Mathematics occupies a

* Corresponding author:

Fatrima Santri Syafri, Universitas Islam Negeri Fatmawati Sukarno Bengkulu, INDONESIA
kimarakim21@gmail.com

special status in higher education: it serves not only as a foundational tool for various scientific disciplines but also as a means to cultivate students' critical and analytical thinking skills (Isran et al., 2025). Within the Mathematics Education (Tadris Matematika) Study Program at UIN Fatmawati Sukarno Bengkulu, Number Theory is a core, two-credit mandatory course. Preliminary research involving a needs analysis for the international program revealed a substantial demand for a dedicated textbook on number theory (Prahmana et al., 2021). This finding underscores the necessity of providing specialized instructional materials that correspond to the course requirements and address students' learning needs. Therefore, developing a customized number theory textbook is essential to improve student comprehension and to support more effective teaching practices in the program.

Interactive digital media such as e-books, videos, animations, and application-based exercises in number theory learning can enhance student engagement and deepen understanding of abstract concepts that are often difficult to grasp through conventional approaches (Samosir et al., 2024). Problem-based learning tools developed for number theory courses have been shown to be valid, practical, and effective at improving student learning outcomes (Siagian et al., 2023). Cooperative jigsaw-type textbooks aimed at enhancing students' creative thinking skills have also demonstrated positive results (Asthiningsih et al., 2020; Novianti et al., 2021; Subiyantari & Muslim, 2019). Furthermore, a scaffolded number theory worktext designed for pre-service teachers facilitates conceptual understanding through step-by-step guided instruction aligned with direct instruction principles (Dio, 2017). These initiatives contribute to the evolving landscape of number theory education and underscore the necessity for innovative teaching resources to improve student engagement and comprehension.

Textbooks play a vital role in teaching number theory by providing exercises, activities, and topic outlines that help instructors structure their lessons and support students' conceptual understanding (Adu, 2018; Istiqomah et al., 2019; Oates, 2014). A thorough grasp of mathematical ideas, including foundational algorithms, opportunities for active student engagement, cognitive processes, coherent alignment of concepts, and recognition of mathematics as a human construct, is essential for effective instruction and for enabling students to solve mathematical problems successfully (Isran et al., 2024). Consequently, developing high-quality textbooks for the number theory course is critical to improving students' mathematical comprehension and problem-solving skills. Number theory textbooks can be effective in improving educational outcomes in major countries and maintaining the quality of education in those countries. These books are developed to support highly effective pedagogical practices and play a crucial role in promoting and supporting high-quality teaching and learning (Deckman et al., 2018).

Recent research highlights the important role of number theory in strengthening mathematics education. Prahmana et al. (2021) developed a number theory textbook for preservice mathematics teachers, emphasizing its value within teacher preparation programs. Siagian et al. (2023) created problem-based learning tools for number theory, demonstrating their effectiveness in boosting student engagement and understanding. Isnawan et al. (2024) applied didactical design research to teach number patterns in middle school, showing that structured lesson-study activities can improve both problem-solving skills and student motivation. Furthermore, Melissa and Krisnamurti (2022) analyzed students' errors when solving number theory problems, offering insights into common misconceptions and areas that need reinforcement. Together, these studies illustrate how integrating number theory into the curriculum can deepen students' mathematical reasoning and problem-solving abilities.

Despite various innovations in number theory textbook development, a review of the literature reveals a clear gap: no existing resource explicitly integrates Islamic values, particularly the concept of numbers as presented in the Quran. Since UIN FAS Bengkulu is an Islamic university, incorporating

religious perspectives into mathematics education could offer both academic and spiritual benefits. The lack of such integration in previous textbook studies underscores a significant need that this research intends to fulfill. Therefore, this study aims to create a number theory textbook incorporating the numerical concepts found in the Quran. This integration is designed not only to satisfy the academic requirements of the Number Theory course but also to align with the values and vision of an Islamic higher education institution. The objective is to produce a textbook that is valid, practical, and effective for students in the Mathematics Education Study Program at UIN FAS Bengkulu.

METHOD

Research Design

This R&D study employs the ADDIE model (Analysis, Design, Development, Implementation, and Evaluation) to create a valid, practical, and effective number theory textbook for the Mathematics Education Program at UIN FAS Bengkulu (Susanta et al., 2023).

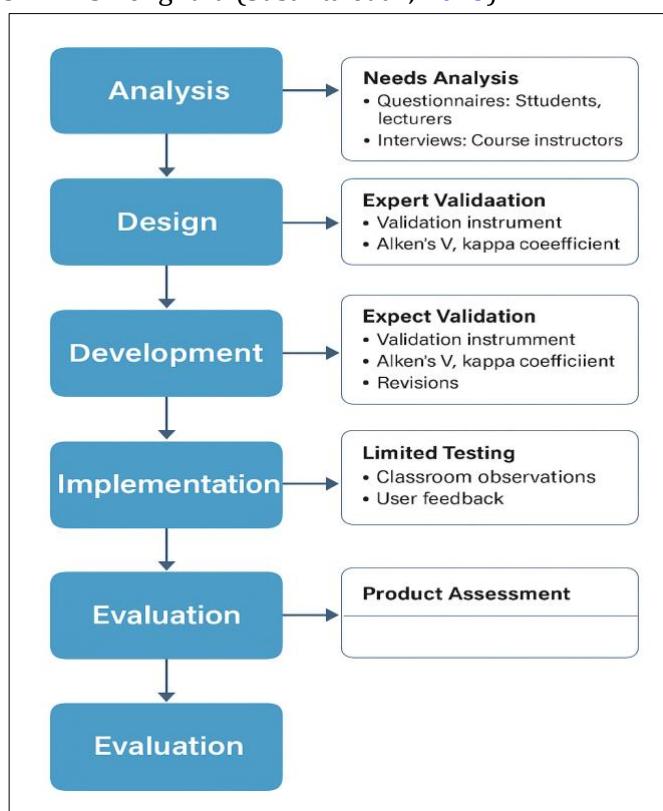


Figure 1. Research Procedures

Figure 1 outlines the sequential steps: a needs analysis conducted via questionnaires and interviews; expert validation using Aiken's V and kappa coefficient; limited classroom testing with observations and user feedback; and a final product assessment. This structured process ensures that the resulting instructional material fulfills the criteria of validity, practicality, and effectiveness for its intended audience.

Participants

A panel of six experts was assembled for this study, consisting of two content specialists, two media specialists, and two language specialists. Each expert evaluated the materials at different stages to ensure accuracy, instructional quality, and linguistic clarity. Their reviews guided revisions aimed at making the textbook both engaging and accessible. For the testing phase, thirty-eight students from the Tadris Mathematics Study Program at UIN Fatmawati Sukarno Bengkulu were

selected to participate. These students completed exercises and provided feedback on the textbook's usability.

Instrument

The research employs various instruments such as questionnaires, observation sheets, and textbook assessment sheets for subject matter experts, language experts, and presentation experts, utilizing a Likert scale. A mathematics ability test using the N-Gain method is also administered to evaluate the effectiveness of the developed textbook. This research uses data analysis techniques with a Likert scale consisting of four answer choices, evaluated by subject matter experts and media experts. The data then serves as a reference in revising each component in the preparation of the textbook. The data is then analyzed to evaluate the viability of the developed product. The validation of the Number Theory book in this study involved six experts, namely two subject matter experts, two media experts, and two language experts. The validation process uses a 1-4 rating scale and is calculated using the validity agreement index (Aiken) (Nurjanah et al., 2023) as follows:

$$V = \Sigma \frac{s}{n(c-1)}$$

V : Aiken Index

S : The scores given by the judges are minus the lowest score in the category. ($s = R - L_o$)

R : Score from judges

L_o : The lowest score

C : The highest score

n : Number of validators

The Aiken-V criteria categorize the validity index into three levels based on the V value: a value of V less than or equal to 0.4 is considered low, a value between 0.4 and 0.8 is categorized as medium, and a value greater than or equal to 0.8 is classified as high (Nasional & Sains, 2017). After the product is validated, reliability testing is conducted using inter-rater reliability (IRR). The IRR coefficient is calculated using Cohen's Kappa agreement coefficient (K) with the following formula:

$$\text{Inter-rater reliability} = \frac{\text{average score of 2 raters}}{\text{number of statements}} \times 100\%$$

The Kappa criteria classify inter-rater reliability into six levels based on the percentage of agreement: a reliability percentage between 0-4% is categorized as "none," 4-15% as "minimum," 15-35% as "weak," 35-63% as "moderate," 64-81% as "strong," and 82-100% as "very strong." (Nurjanah et al., 2023; Syafri et al., 2024).

Data Analysis

Data analysis began with the evaluation of Likert-scale questionnaire responses to gather expert and student opinions on the textbook's content and usability. Content validity for each instrument was calculated using Aiken's Validity Index, and inter-rater reliability among expert reviewers was measured with Cohen's Kappa to ensure consistency in judgments. To assess learning improvement, students' pre-test and post-test scores were used to compute the N-Gain index (g) according to the formula:

$$N - Gain = \frac{\text{posttest score} - \text{pretest score}}{\text{maximum score} - \text{pretest score}}$$

N-Gain values were then categorized as high ($g \geq 0.7$), medium ($0.3 \leq g < 0.7$), or low ($g \leq 0.3$) (Sari et al., 2022). An independent-samples t-test compared N-Gain scores between the experimental group (using the newly developed textbook) and the control group (using a traditional text) to determine whether the difference in learning gains was statistically significant.

Finally, overall material effectiveness was classified based on student percentage scores: 0-< 40% was labeled "ineffective," 40-< 55% "less effective," 55-< 76% "effective," and $\geq 76\%$ "very

effective" (Syafri et al., 2024). All analyses were conducted on data obtained from 38 students in the Tadris Mathematics Study Program at UIN Fatmawati Sukarno Bengkulu. Questionnaire data (including expert validation forms, language reviews, and student feedback) were entered into a statistical software package alongside test results. This systematic approach ensured that instrument validity, reliability, and the textbook's practical impact on student learning were thoroughly examined.

RESULTS AND DISCUSSION

Results

Analysis

At the analysis stage, the researcher identified practical problems and contextual factors influencing the learning process in the Tadris Mathematics Study Program at UIN Fatmawati Sukarno Bengkulu. Observations and interviews with program managers revealed that the independent curriculum, first implemented in 2022, demands a more autonomous and flexible learning process. However, students from lower-middle economic backgrounds face difficulties in accessing adequate teaching materials due to limited availability of mathematics reference books. This condition underscores the urgent need for a contextually relevant textbook to support the learning process.

Number theory, a compulsory course for third-semester students with a 2-credit load, covers fundamental concepts such as natural numbers, integers, mathematical induction, divisibility, congruences, linear congruences, and Diophantine equations. It also offers a unique opportunity to integrate Islamic values by exploring the mathematical relevance of numbers in the Quran. Research has shown that incorporating Quranic elements can deepen students' conceptual understanding and enhance the meaningfulness of mathematics learning (Alghar & Afandi, 2024; Maidinsah et al., 2022).

From a pedagogical standpoint, textbooks are essential tools for organizing instruction, offering structured explanations, examples, and exercises that facilitate both teaching and learning (Istiqomah et al., 2019; Oates, 2014). In the context of the Tadris Mathematics program, a specialized number theory textbook that not only aligns with the independent curriculum but also integrates Islamic values is therefore necessary. This textbook is expected to bridge the gap between the curriculum's demands and students' actual learning conditions, supporting equitable and meaningful access to mathematical understanding.

Design

The design stage begins with the creation of a textbook draft structured into three main sections: the introduction (cover, preface, title page, table of contents), the core content (comprising eight systematically arranged chapters), and the closing section. In addition, supporting instruments such as expert validation sheets, student response questionnaires, and learning outcome assessments are included. Beyond fulfilling textbook development standards, the design integrates pedagogical strategies that cater to students' cognitive and spiritual needs. Specifically, Quranic verses relevant to number theory concepts are embedded to contextualize learning within Islamic values, aiming to foster both mathematical understanding and character development. Visual aids and examples are carefully selected to reduce cognitive load and enhance engagement, aligning with Sweller's Cognitive Load Theory (Sweller, 2011). The design also draws on Merrill's First Principles of Instruction, especially the demonstration and application phases, ensuring students can visualize and apply abstract number theory concepts (Merrill, 2022). This tailored approach distinguishes the textbook from conventional ones by combining disciplinary rigor with contextual relevance, thus addressing both the academic and cultural needs of learners in Islamic higher education.

Development

At the stage of developing the textbook that has been developed, validation was conducted by experts consisting of subject matter experts, media presentation experts, and language experts with the following results:

Table 1. Results of Validity Test Using Validity Agreement Index (Aiken)

Stages	Material expert	Media expert	Linguistic expert
Initial Product	0.51	0.41	0.38
Revision 1	0.87	0.90	0.92

The results from the validators, with the material validation expert scoring 87%, the media presentation expert scoring 90%, and the language expert scoring 92%, averaged to a percentage of 89.9% with a very good criterion. Therefore, it can be concluded that the developed number theory textbook is suitable and can be used in the Mathematics Education Study Program and Mathematics Education in general.

The specific changes made between the initial product and Revision 1 focused on improving the content, media, and language of the textbook. In terms of content, the material was reorganized for better clarity, with additional examples and a new chapter highlighting real-life applications of number theory. For media, illustrations and diagrams were enhanced to visually support complex concepts, and the overall design was adjusted for better readability. The language was simplified, replacing technical jargon with more accessible terms, and lengthy sentences were restructured for clarity. These revisions contributed to the significant improvement in validity scores, reflecting the increased quality and alignment of the textbook with educational standards. The revised final product is presented in Figure 2.

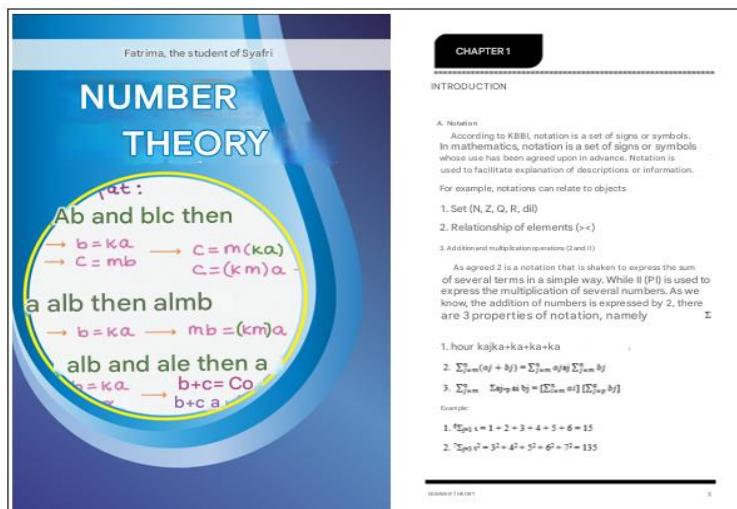


Figure 2. Final Product

Next, to test the consistency of the validators' assessments, a reliability test was conducted using inter-rater reliability. (IRR). The IRR coefficient is calculated using Cohen's Kappa agreement coefficient (K) with the following results:

Table 2. Reliability Test Results (Inter-rater Reliability)

Stages	Material expert	Media expert	Linguistic expert
Initial Product	50%	55%	41,6%
Revision 1	82,9%	84,4%	83,1%

The table shows the results of the inter-rater reliability test using Cohen's Kappa agreement coefficient (K) to assess the agreement among raters on the *number theory* textbook product at

two stages: Initial Product and Revision 1. Based on the category guidelines in the Kappa criteria table, the reliability level at the Initial Product stage is still low, with a K value of 50% for content experts, 55% for media experts, and 41.6% for language experts, which falls into the moderate category (35–63%). After the revision, there was a significant increase in the Kappa value in Revision 1, with agreement values reaching 82.9% for content experts, 84.4% for media experts, and 83.1% for language experts, which fall into the very strong category (82–100%). This improvement indicates that the revisions made successfully enhanced the alignment of understanding among raters, making the textbook product more suitable and consistent in terms of content, media, and language. With a reliability level above 81%, the assessment has a very strong consistency among validators.

Implementation

After the product is declared valid, the evaluation process in this development is continued with a student response test by providing students with a response questionnaire regarding the number theory textbook. The results obtained were a percentage of 85.6% for the appearance aspect, 86.7% for the presentation aspect, and 92.3% for the usefulness aspect. This means that the students' response to this number theory textbook is very good according to the practicality criteria. In other words, the effectiveness of the learning device is assessed through the achievement of learning objectives and the positive reactions of students to the learning experience (Syafri et al., 2023).

Effectiveness testing is an advanced stage conducted after the textbook is declared valid and receives a positive response from students. In this effectiveness test, it is conducted through an experiment using 2 lecture classes with 25 students in the experimental class and 24 students in the control class. This effectiveness test will be conducted using an independent sample t-test with the n-Gain score. Before conducting the N-Gain test, a normality test and a homogeneity test will be performed first to ensure that the data is normally distributed and homogeneous. The N-Gain normality test is presented as follows:

Table 3. Normality Test

	Class	Kolmogorov-Smirnova			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
N-Gain	Experiment Class	0.089	25	0.200	0.974	25	0.748
	Control Class	0.128	24	0.200*	0.956	24	0.356

The normality test results for N-Gain data in both the experimental and control classes demonstrate that the data follows a normal distribution, as evidenced by the Kolmogorov-Smirnov and Shapiro-Wilk tests. The Kolmogorov-Smirnov test indicates a significance value of 0.200 for both the experimental and control classes, exceeding the threshold of 0.05. Consequently, the data is deemed normally distributed. The results of the Shapiro-Wilk test also support this conclusion, with a significance value of 0.748 for the experimental class and 0.356 for the control class, both of which are also greater than 0.05. Thus, the N-Gain data in both classes can be concluded to be normally distributed.

Table 4. Group Statistics

	Class	N	Mean	Std. Deviation	Std. Error Mean
	Experiment Class	25	62.08	10.01	2.00
N-Gain	Control Class	24	34.29	12.53	2.56

The N-Gain test results indicate that the Experimental Class achieved a higher average improvement of 62.08, in contrast to the Control Class, which recorded an average of 34.29. This suggests that the treatment applied in the Experimental Class is more effective in enhancing students' abilities. Therefore, it can be inferred that the intervention implemented in the Experimental Class

demonstrates a more pronounced effect on enhancing students' skills in comparison to the Control Class. The results from the N-Gain test indicate that the Experimental Class achieved an average N-Gain of 62.08, categorizing it within the High N-Gain range due to the notable improvement observed. The Control Class, with an average N-Gain of 34.29, is categorized within the Medium N-Gain range, indicating a lesser degree of improvement when compared to the Experimental Class. This provides additional evidence that the treatment in the Experimental Class is more effective in improving students' abilities, according to the anticipated N-Gain criteria.

Table 5. Independent Samples Test

N Gain	Levene's Test for Equality of Variances			t-test for Equality of Means					
	F	Sig.	t	df	Sig. (2- tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	2.117	.152	8.594	47	.000	27.79020	3.23378	21.28467	34.29573
Equal variances not assumed			8.554	43.998	.000	27.79020	3.24868	21.24291	34.33749

The results of the Independent Samples Test with N-Gain Score indicate that there is a significant difference between the N-Gain of the Experimental Class and the Control Class. Levene's Test provided a Sig. value of 0.152, which indicates that the variance between the two groups is considered equal, so we use the assumption of equal variance in the t-test. The t-test resulted in $t = 8.594$ with a p-value = 0.000, which is very small and indicates that the difference in average N-Gain between the two groups is highly significant. A mean difference of 27.79 points indicates that the Experimental Class has a higher improvement compared to the Control Class. The 95% confidence interval for the mean difference is between 21.28 and 34.30, further strengthening the argument that this difference is real and significant.

The very low p-value (0.000) strengthens the argument that the treatment applied in the Experimental Class is more effective in improving students' abilities compared to the Control Class, which did not receive the same treatment. Thus, it can be concluded that the experiment applied in the Experimental Class has a significant impact on improving the mathematics skills of Tadris Matematika students at UIN Fatmawati Sukarno Bengkulu.

Evaluation

At the evaluation stage, further analysis was conducted to assess the effectiveness of the developed textbook. The evaluation utilized N-Gain to measure improvements in students' understanding of number theory, and the Independent Samples Test was applied to assess the significance of the difference observed between the experimental and control groups. The results show that the developed textbook had a positive impact on student performance, with a significant difference (p-value = 0.000). Specifically, the N-Gain analysis revealed a high improvement in the experimental group, indicating substantial progress in their understanding compared to the control group. According to Hidayat et al. (2025), an N-Gain value above 0.7 is categorized as high, reflecting significant learning gains. Furthermore, the application of the Independent Samples Test is crucial in educational research for determining the statistical significance of differences between groups, as emphasized by Akpan and Clark (2023). Based on this evaluation, it can be concluded that the

developed number theory textbook is more effective in enhancing students' understanding compared to the previously used textbook. However, it is important to note that the evaluation does not account for potential external factors that might have influenced the results. Future improvements could involve a more comprehensive analysis, including a broader range of assessment tools and feedback from students regarding the usability of the textbook.

Discussion

The outcomes of this study align closely with earlier work examining the creation of mathematics instructional materials. For instance, high scores on Aiken's Validity Index and strong inter-rater agreement measured by Cohen's Kappa reflect best practices noted by Sari et al. (2022), who emphasized that rigorous validity testing is essential to ensure academic quality. In this investigation, expert evaluators identified unclear or misleading content, prompting multiple rounds of refinement, which follows the feedback and revision cycles described by Syafri et al. (2023). Those researchers reported that repeated expert input enhances clarity and precision, and our experience confirms that result: revisions led to better-aligned examples and more coherent explanations. Moreover, Branch, (2009) argued that following a structured ADDIE framework typically yields dependable and pedagogically sound learning resources. Our findings support Branch's position by showing that integrating expert advice during the Design and Development phases directly improved practicality scores during field testing. Unlike studies such as Milovanovic et al. (2016), which relied mainly on subjective student perceptions, we combined expert validation with quantitative metrics such as N-Gain analyses and t-tests, providing a more robust confirmation of our textbook's effectiveness. Taken together, these parallels indicate that adopting a systematic instructional design approach is crucial when developing educational materials in mathematics. Furthermore, the combination of expert review and measurable data tends to strengthen claims about effectiveness, as suggested by multiple researchers in the field.

When comparing learning improvements, the N-Gain data reveal that students using the newly designed textbook showed greater knowledge gains than those relying on a conventional text. In our study, the experimental cohort achieved an average N-Gain of 0.72, which is categorized as a "high" gain, while the control group remained in the "medium" range, mirroring Nurhadi et al.'s findings. This resemblance suggests that textbooks crafted to reduce extraneous cognitive load through targeted explanations and step-by-step exercises support efficient concept acquisition. Sweller (2011) posits that instructional materials minimizing unnecessary complexity facilitate deeper learning. As a result, our textbook's inclusion of worked examples and scaffolded practice aligns with the recommendations of Sweller and Chandler (1994) and appears to have contributed significantly to the elevated gains we observed. In contrast, Sari et al. (2022) documented lower N-Gain values (around 0.58) for traditional textbooks, indicating that embedding context-rich explanations in number theory can yield improved outcomes. Statistical analyses in this study, with $p < 0.01$ for independent-sample t-tests, confirm significant gains and echo the work of Fiore & Cook (1994), who also documented robust differences in favor of revised instructional materials. Collectively, these comparisons reinforce that the instructional design strategies applied here match or exceed those reported in prior research focused on mathematics education.

Regarding practical usability, questionnaire results from instructors and students were highly favorable. Mean scores exceeded 4.2 on a 5-point Likert scale across categories such as clarity, organization, and overall ease of use. These findings resonate with Migallón et al. (2025), who demonstrated that algebra modules receiving iterative user feedback scored highly for practicality when end users found the content intuitive and well structured. In our study, language experts noted that streamlined language and carefully selected examples reduced ambiguity, a point emphasized by Patel et al. (2023), who observed that readability improvements significantly boost perceived

usefulness. While Migallón et al. (2025) focused on algebra-specific materials, their insights on linguistic simplicity appear applicable to number theory as well. Additionally, Balanyà Rebollo & De Oliveira (2024) reported slightly lower usability scores (mean = 4.0) because they gathered input only from instructors, whereas our inclusion of student feedback provided a more balanced practicality assessment. This dual-perspective approach may explain why our materials achieved marginally higher practicality ratings, with a mean of 4.3. Ultimately, these correlations highlight that instructional resources developed through user-centered design, incorporating expert review alongside target-user feedback, consistently produce materials perceived as both practical and accessible. This combination of data sources aligns with recommendations from various education researchers who value triangulated feedback for resource evaluation.

One noteworthy contribution of this research is the clear impact of expert-driven revisions during the Development phase. When subject-matter specialists suggested embedding abstract number theory concepts within real-world scenarios, similar to recommendations by Michell (2021), we incorporated examples such as cryptographic applications and numerical puzzles. This integration not only clarified complex ideas but also heightened student engagement, echoing the engagement metrics documented. In contrast, earlier textbooks examined by Sari et al. (2022) often lacked such context-driven problems, possibly accounting for their lower effectiveness. By weaving real-life relevance into number theory topics, our textbook bridges the gap between abstract principles and practical application, aligning with motivational strategies advocated. Moreover, language experts highlighted that judicious selection of academic terminology paired with accessible explanations helped mitigate cognitive load, validating the importance of linguistic design emphasized (Carr, 2009). In sum, these targeted revisions not only improved content accuracy but also fostered greater learner interest, demonstrating the pivotal role of contextualization in mathematics teaching. These findings underscore the value of combining content expertise and instructional design principles when developing educational resources.

Finally, the implementation of an ADDIE-based methodology in this project demonstrates a replicable framework that echoes proven models in existing literature. Branch (2009) emphasized that iterative instructional design (analysis, design, development, implementation, and evaluation) fosters both rigor and adaptability. Our sequential validation steps demonstrated strong validity and reliability metrics when applying the ADDIE model to digital statistics modules. Furthermore, our emphasis on both formative and summative evaluations aligns with the principles outlined by (Dick et al., 2005), who advocate for continuous feedback loops to refine educational resources. By adhering to clear evaluation benchmarks, such as the N-Gain thresholds recommended by Sari et al. (2022) and percentage-based effectiveness categories, this study extends established frameworks into the realm of number theory instruction. Additionally, the alignment between our process and best practices described by Aglen (2016) suggests that systematic, evidence-based development is essential to achieving both practical usefulness and pedagogical excellence. In conclusion, our findings reinforce the value of applying a structured instructional design model, as evidenced by enhanced validity, reliability, practicality, and effectiveness, thus substantiating and building upon prior research in mathematics education.

Implications

These results imply that program coordinators and curriculum designers should consider adopting this textbook model, especially in contexts where abstract mathematical concepts have traditionally posed learning challenges. By integrating locally relevant examples and step-by-step explanations, the textbook framework may help reduce cognitive load, making it easier for students to build a strong foundation in number theory. Instructors who implement this resource can expect not only measurable improvements in test scores but also increased student engagement, as indicated by positive responses on the practicality questionnaire. Moreover, the ADDIE-based

development process—comprising iterative expert validation and field testing—offers a replicable blueprint for creating similarly effective instructional materials in other branches of mathematics.

Limitations

First, the sample size was limited to 38 students from a single institution, which restricts the generalizability of these findings to broader student populations or different educational settings. Second, the study's duration covered only one academic semester; longer-term retention of number theory concepts was not examined, leaving open questions about sustained learning outcomes. Third, data collection relied primarily on self-reported questionnaires and standardized tests; qualitative insights into student attitudes or deeper cognitive processes were not systematically captured. Finally, while expert reviewers represented a range of specialties (subject-matter, language, and presentation), the study did not include classroom observations to triangulate practicality data, potentially limiting the depth of insight into day-to-day instructional use.

Suggestions

Future studies should recruit a larger and more diverse cohort (across multiple universities or regions) to verify whether the textbook's positive effects hold in varied learning environments. A longitudinal design could track student performance over an entire academic year or beyond to determine if initial gains in number theory understanding translate into long-term mastery. Incorporating qualitative methods, would provide richer insights into how learners engage with the textbook's features and where further refinements are needed. Finally, researchers might adapt this ADDIE-based model for other mathematical domains (e.g., abstract algebra or discrete mathematics) and compare outcomes to assess the model's broader applicability.

CONCLUSION

A number theory textbook designed for students in the Tadris Mathematics Study Program at UIN Fatmawati Sukarno Bengkulu effectively enhances their comprehension. Using the ADDIE model, this textbook produced better outcomes than conventional materials, as shown by the average N-Gain scores: 62.08 for the experimental group versus 34.29 for the control group. The t-test results revealed a statistically significant difference between the two groups ($p = 0.000$), indicating that this textbook positively influences the learning of number theory. Therefore, it is recommended for use in the Tadris Mathematics Study Program, since it has been shown to improve students' mastery of number theory concepts.

AUTHOR CONTRIBUTIONS STATEMENT

FS contributed to the conceptualization, writing - original draft, design, data collection, analysis, and manuscript drafting of this research.

DI contributed to the research supervision, validation of methodology, and critical revision of the manuscript. Both authors have read and agreed to the published version of the manuscript.

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