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Symmetries of the equilateral triangle mediated by geogebra: a proposal for training rural mathematics teachers

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Abstract

Background: The equilateral triangle is a fundamental object in geometry, notable for its symmetrical properties, which are best understood through dynamic visualization. Traditional methods often fall short in making these concepts accessible, especially in rural educational contexts with limited resources.
Aims: This study aimed to explore the effectiveness of GeoGebra software as a mediating tool to enhance rural mathematics teacher training, particularly in understanding the symmetries of equilateral triangles.
Methods: Employing a qualitative and descriptive methodology, the research collected data through classroom observations, analysis of participants' digital constructions using GeoGebra, and reflective interviews. These methods were chosen to capture how pre-service teachers engage with geometric concepts via dynamic tools.
Results: The results demonstrated that GeoGebra significantly improves the visualization of geometric transformations such as rotations and reflections. Participants reported increased comprehension and engagement due to the interactive nature of the software. The teaching approach also encouraged deeper geometric reasoning and fostered the ability to connect abstract group theory with concrete visual representations.
Conclusion: The use of GeoGebra in rural teacher training is not only feasible but transformative. As a free, accessible platform, it bridges gaps caused by resource limitations and aligns well with local educational realities. The software empowers future educators to adopt student-centered approaches that make geometry both meaningful and contextually relevant. This proposal supports the integration of digital tools in teacher education and provides a replicable model for similar contexts globally. Future research should investigate broader applications of GeoGebra across other geometric topics and assess its long-term impact on classroom practice in rural environments.

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INTRODUCTION

Geometry plays a fundamental role in mathematics education as it cultivates spatial reasoning and logical thinking among students. Yet, its abstract nature makes it challenging, especially in rural educational contexts with limited resources and didactic tools. In such settings, mathematics teachers often struggle to convey geometric concepts in an engaging and comprehensible manner. Integrating digital tools into mathematics instruction has emerged as a promising approach to address these pedagogical gaps. GeoGebra, a dynamic and open-access mathematics software, offers features that enable users to visualize geometric transformations interactively. Its potential in improving comprehension of abstract mathematical ideas has been highlighted in studies such as Oke & Aliu (2025) and Putra et al. (2023), which emphasized its usability in teacher training and school-level instruction. For rural areas, where technology integration is still underdeveloped, adopting user-friendly tools like GeoGebra is particularly urgent. This study explores how GeoGebra

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can support the teaching of geometric symmetry, with a specific focus on equilateral triangles, to future mathematics teachers in rural environments.

GeoGebra serves not only as a visualization aid but also as a cognitive scaffold that enhances mathematical reasoning. Through its integration in pre-service teacher training, GeoGebra has been shown to develop both content mastery and digital teaching competence. In rural contexts, equipping teachers with such competencies is essential for bridging the urban-rural educational divide. Moreover, the exploration of symmetry—especially in objects like equilateral triangles—offers a concrete pathway to connect geometry with group theory. According to Oke & Aliu (2025) and Putra et al. (2023), GeoGebra-assisted lessons allow pre-service teachers to internalize geometric properties through trial, manipulation, and reflection. This active learning model is especially relevant for future educators expected to teach with minimal physical resources. By implementing such software in teacher education, we anticipate improved instructional quality and increased student engagement in geometry classes. Therefore, this study positions itself at the intersection of digital learning tools and rural teacher preparation.

The concept of symmetry has long been recognized as a gateway to more complex mathematical reasoning. Classical group theory, as introduced by Galois and expanded by Klein and Weyl, provided a framework for understanding transformations and invariance in both algebra and geometry (Klein 1893; Weyl 1952; Wussing 1984). The equilateral triangle, due to its highly regular structure, serves as an exemplary model of geometric symmetry. It exhibits three lines of reflective symmetry and three rotational symmetries, which together form the dihedral group D3—a foundational example of non-abelian finite groups (Armstrong, 1988). Digital tools like GeoGebra allow these concepts to be explored in real time, moving them beyond theoretical abstraction into visual experimentation. Studies by Kovács et al. (2023) and Vedrenne-Gutiérrez et al. (2021) confirm that such dynamic representations enhance learner understanding and foster mathematical curiosity. In this context, training rural teachers using GeoGebra can yield pedagogical models that are both accessible and effective. This justifies the investigation of GeoGebra's utility for teaching symmetries in equilateral triangles within pre-service training programs.

Moreover, rural schools often face the double burden of inadequate infrastructure and a lack of qualified teachers trained in contemporary pedagogical methods. The integration of accessible educational software like GeoGebra can mitigate these challenges by providing low-cost, high-impact instructional solutions. Hall & Bastos. (2024) demonstrated how GeoGebra projects involving string art enabled students to explore patterns and symmetry creatively. Chehlarova & Gachev (2023) also noted increased engagement and conceptual depth among learners using digital symmetry tasks within STEAM-oriented learning. These findings support the hypothesis that GeoGebra not only aids in concept delivery but also enhances learners' motivation and autonomy. Additionally, in teacher education, digital simulations can serve as safe spaces for pedagogical experimentation before real classroom application. Given the socioeconomic and technological barriers often found in rural education, this study underscores the urgency of including digital geometry tools in teacher preparation curricula. By focusing on a familiar yet rich concept like triangle symmetry, we aim to show how even basic geometric topics can benefit from technological enhancement. Ultimately, this reinforces the value of digital equity and pedagogical innovation in rural mathematics education.

The urgency of integrating digital tools into teacher education arises from the need to ensure equitable access to quality mathematics instruction across different educational contexts. Rural schools, in particular, often suffer from a lack of updated pedagogical resources and technological infrastructure. This situation necessitates the adoption of flexible, cost-effective solutions that can enhance teaching and learning experiences. GeoGebra emerges as an ideal candidate due to its free access, intuitive interface, and proven effectiveness in mathematical instruction. Its integration into teacher training has been underutilized, especially for preparing educators who will teach in rural or resource-constrained environments. This study was conceptualized to respond to that gap by evaluating the role of GeoGebra in helping teachers understand and teach symmetry through the lens of the equilateral triangle. By doing so, we aim to empower future educators with both the content knowledge and digital fluency necessary for modern classrooms. The broader rationale is to improve mathematics education quality by equipping teachers with tools that align with 21st-century learning goals.

The foundations of group theory date back to the 19th century with Galois' groundbreaking work, which linked polynomial solvability with group structures, establishing group theory as a central concept in mathematics (Wussing, 1984). Later, Cayley formalized group concepts abstractly, and Klein's Erlangen Program revolutionized geometry by proposing that geometric properties should be understood through group-preserving symmetries (Klein, 1893). Weyl's exploration of symmetry groups further expanded their relevance, noting that these structures underpin not only mathematical entities but also phenomena in nature and the arts (Weyl, 1952). These classical ideas remain central in modern mathematical education, particularly in geometric reasoning. The equilateral triangle, with its six distinct symmetries—three rotations and three reflections—forms the dihedral group D3, which serves as a tangible entry point to explore group structures (Armstrong. 1988). Recent pedagogical shifts emphasize the role of digital tools like GeoGebra in visualizing such mathematical abstractions. For example, Vedrenne-Gutiérrez et al. (2021) demonstrated how e-book-based tasks using GeoGebra enhanced primary school students' conceptual understanding of symmetry through interactive media. Likewise, Kovács et al. (2023) showed that combining GeoGebra with computer algebra systems enriched learners' abilities to test and visualize conjectures, making abstract topics more concrete and assessable.

In the context of educational innovation, GeoGebra has gained attention as a bridge between theoretical mathematics and interactive learning. Hall & Bastos. (2024) employed GeoGebra to explore symmetry through artistic constructs, revealing its capacity to connect mathematical content with real-life and aesthetic domains. Similarly, Chehlarova & Gachev (2023) presented case studies from STEAM-based mathematics education, illustrating how dynamic software facilitated deeper student involvement in geometry. Such findings resonate with the need for more accessible and engaging approaches to teaching symmetry, especially for novice learners and teacher trainees. In rural education environments, where traditional resources are scarce, the integration of free, open-access tools like GeoGebra becomes increasingly relevant. The pedagogical potential of these tools lies not only in enhancing visualization but also in fostering reasoning and self-directed exploration of geometric relationships. As supported by Hershkovitz et al. (2023), learners demonstrated higher engagement levels when using feedback-enhanced digital tasks within symmetry-related topics. These recent advancements underline the value of incorporating GeoGebra into teacher training programs, particularly to reinforce foundational concepts like symmetry in a dynamic and exploratory way. Thus, integrating these findings with historical frameworks of group theory provides both the theoretical depth and the applied innovation required for impactful geometry instruction in contemporary classrooms.

Although many studies have confirmed the pedagogical benefits of using GeoGebra in mathematics education, few have specifically focused on rural teacher training programs. Most existing research concentrates on student performance and technology integration in urban or well-resourced settings. There is limited evidence on how pre-service teachers in rural contexts experience and implement GeoGebra-based learning. Additionally, while concepts like symmetry and transformation are commonly addressed in the curriculum, their connection to abstract algebra through group theory is often neglected. Even fewer studies have contextualized the learning of equilateral triangle symmetry within rural education frameworks. Moreover, the potential of GeoGebra to bridge the gap between visual and algebraic thinking remains underexplored in teacher education. This study addresses those omissions by focusing specifically on symmetry learning through GeoGebra in a rural teacher training context. In doing so, it aims to contribute to both practice and theory in mathematics education reform.

The primary objective of this study is to explore the effectiveness of GeoGebra as a digital tool for teaching the symmetries of the equilateral triangle to pre-service mathematics teachers in rural areas. The study hypothesizes that interactive visual tools enhance conceptual understanding and pedagogical confidence. More specifically, it seeks to investigate whether GeoGebra can foster deeper reasoning, improved visualization, and stronger links between geometry and algebra. It also aims to determine the extent to which GeoGebra can serve as a bridge for teaching mathematical abstraction in under-resourced settings. Through a qualitative lens, the research will analyze teachers' interactions with the software, the accuracy of their constructions, and their reflective responses. A secondary goal is to identify best practices for integrating GeoGebra into teacher training programs. This includes recommendations for curriculum design and professional development in digital pedagogy. Ultimately, the study aspires to inform policy and improve equity in mathematics education through technology-enabled teaching innovations.

METHOD

Research Design

This study employs a qualitative and descriptive research design to investigate the impact of GeoGebra on the teaching of symmetries in equilateral triangles. A qualitative approach allows for an in-depth understanding of teacher experiences and the dynamics of integrating technology into pedagogical practice. The focus is on observing, interpreting, and contextualizing how pre-service rural mathematics teachers engage with symmetry through digital tools. The descriptive component seeks to outline observable instructional behaviors and interactions with GeoGebra, without aiming for statistical generalization. This method is suitable for exploring educational interventions that are context-specific, such as rural teacher training with limited infrastructure. The integration of qualitative and descriptive elements offers a balanced view of both process and outcomes. As Miftach Fakhri et al. (2025) ansd Yorganci & Subasi, (2025) demonstrated, digital tools like e-book tasks using GeoGebra foster nuanced engagement with geometric concepts. This study builds on such frameworks by examining digital mediation in a setting often underrepresented in educational research.

Participant

The participants of this study consisted of pre-service mathematics teachers enrolled in a teacher training program located in a rural educational institution. A total of 15 participants were purposively selected based on their availability and willingness to integrate GeoGebra into their geometry lessons. The demographic focus on rural participants addresses a critical equity gap in access to technology-based pedagogy. All participants had basic computer literacy but limited experience with dynamic geometry software prior to the study. By selecting individuals from a rural context, the research investigates how digital transformation can be made inclusive in low-resource environments. According to Jesionkowska et al. (2020) and Joglar Prieto et al. (2014) GeoGebra activities in early education have shown promise in stimulating geometric thinking even among novice users, validating the choice of novice teachers as participants. This aligns with the broader aim of empowering future educators to leverage technology despite infrastructural constraints. The participants also represented diverse academic backgrounds, contributing to a richer dataset for interpreting engagement patterns.

Instruments

Data were collected through three primary instruments: classroom observation protocols, digital screen recordings of GeoGebra use, and semi-structured interviews. Observational data focused on how participants used GeoGebra to construct, manipulate, and explain the symmetries of equilateral triangles. Screen recordings captured real-time interactions, such as point reflections and rotational commands, providing insight into cognitive engagement with the software. Interview responses explored participants' perceptions, difficulties, and reflections on their learning experiences. This multi-instrument approach ensures triangulation, enhancing the reliability of the qualitative findings. The selection of instruments was informed by Oner (2016) and Schwarz et al. (2018), who employed digital logs and feedback cycles in analyzing learners' construction of geometric conjectures using GeoGebra. The instruments were piloted with a subset of participants to ensure clarity and usability. All data were anonymized to maintain ethical research standards.

Data Analysis

Data analysis followed a thematic coding strategy, allowing recurring patterns, behaviors, and themes to emerge from the qualitative dataset. Transcripts from interviews and video recordings were transcribed and coded manually using inductive category development. The process was iterative, involving multiple rounds of refinement to ensure representational accuracy. Observational data were organized into episodes of instructional activity, focusing on the visual construction and articulation of symmetries. Inspired by Happel et al. (2014) and Hershkovitz et al. (2024), who studied behavior patterns in feedback-enhanced digital symmetry tasks, this study paid particular attention to moments of conceptual breakthrough or confusion. Codes were categorized under themes such as "visualization of rotation," "reflections understanding," and "GeoGebra navigation." The coded data were then synthesized to form a narrative that captured both the individual and collective learning experiences. Findings were interpreted within the theoretical framework of dynamic geometry learning and teacher cognition.

Symmetry Group of the Equilateral Triangle

An equilateral triangle has 6 symmetries, which can be described as follows:

(i) Identity: This is the symmetry that does not alter the triangle. It is as if we did nothing to it.

(ii) Rotations: The equilateral triangle has rotational symmetry. We can rotate it around its barycenter (the meeting point of the medians) at two specific angles, in the counterclockwise direction:

120º rotation: By rotating the triangle 120º, it overlaps its original position.

240º rotation: By rotating another 120º (totaling 240º), the triangle also overlaps its original position.

(iii) Reflections: The equilateral triangle also has reflexive symmetry. We can reflect it through three axes of symmetry, which coincide with its medians (line segments that connect a vertex to the midpoint of the opposite side). Each reflection mirrors the triangle in relation to one of these medians (Valladares, 2018).

The combination of these symmetries (identity, rotations and reflections) results in the 6 symmetries of the equilateral triangle. Each of these symmetries corresponds to a permutation of the triangle's vertices. A figure illustrating these symmetries and the corresponding permutations of the vertices would be useful for better visualization.

The symmetries of an equilateral triangle include identity, two rotations (120° and 240°) around the barycenter and three reflections in relation to its medians. Each of these operations corresponds to a permutation of the triangle's vertices.

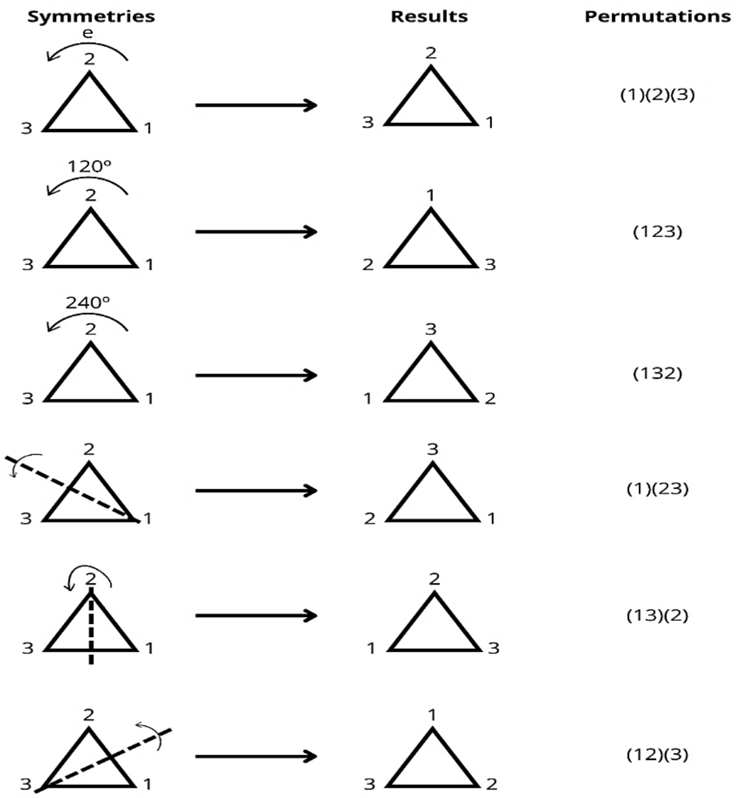

Figure 1. Symmetries of the equilateral triangle

This teaching proposal explores the Symmetries of the Equilateral Triangle of rotation and translation using the GeoGebra software. The goal of GeoGebra is to provide teachers with a visual and interactive understanding of these concepts, connecting geometric intuition with algebraic representation. Rotations involve the movement of an object around a fixed point (center of rotation), while translations consist of the displacement of an object in a specific direction and distance. GeoGebra offers a dynamic environment that facilitates exploration and experimentation with mathematical concepts, allowing subjects to build their own knowledge Hohenwarter et al., 2008).

Rotation:

- Create an object: Draw a point, segment, polygon or any other figure that you want to rotate.
- Select the Rotation tool: Click on the rotation icon (a point rotating around another).
- Select the object to be rotated: Click on the object you drew.
- Select the center of rotation: Click on the point that will be the center of rotation.
- Set the rotation angle: Enter the value of the desired angle (in degrees or radians) and click “OK”.

Translation:

- Create an object: Draw a point, segment, polygon or any other figure that you want to translate.
- Select the Translation by vector tool: Click on the translation icon (an arrow).
- Select the object to be translated: Click on the object you drew.
- Set the translation vector: Click on two points that define the vector (the first point is the starting point of the vector and the second is the end point).

GeoGebra makes it possible to visualize the connection between algebra and geometry. The symmetries of the equilateral triangle, such as rotations and translations, can be represented algebraically by algebraic commands in the construction window. For example, a rotation about the origin in the Cartesian plane can be represented by a rotation matrix.

RESULTS AND DISCUSSION

Results

This algebraic representation allows the precise calculation of the coordinates of the transformed points. Algebra provides a language for describing and generalizing geometric relationships.

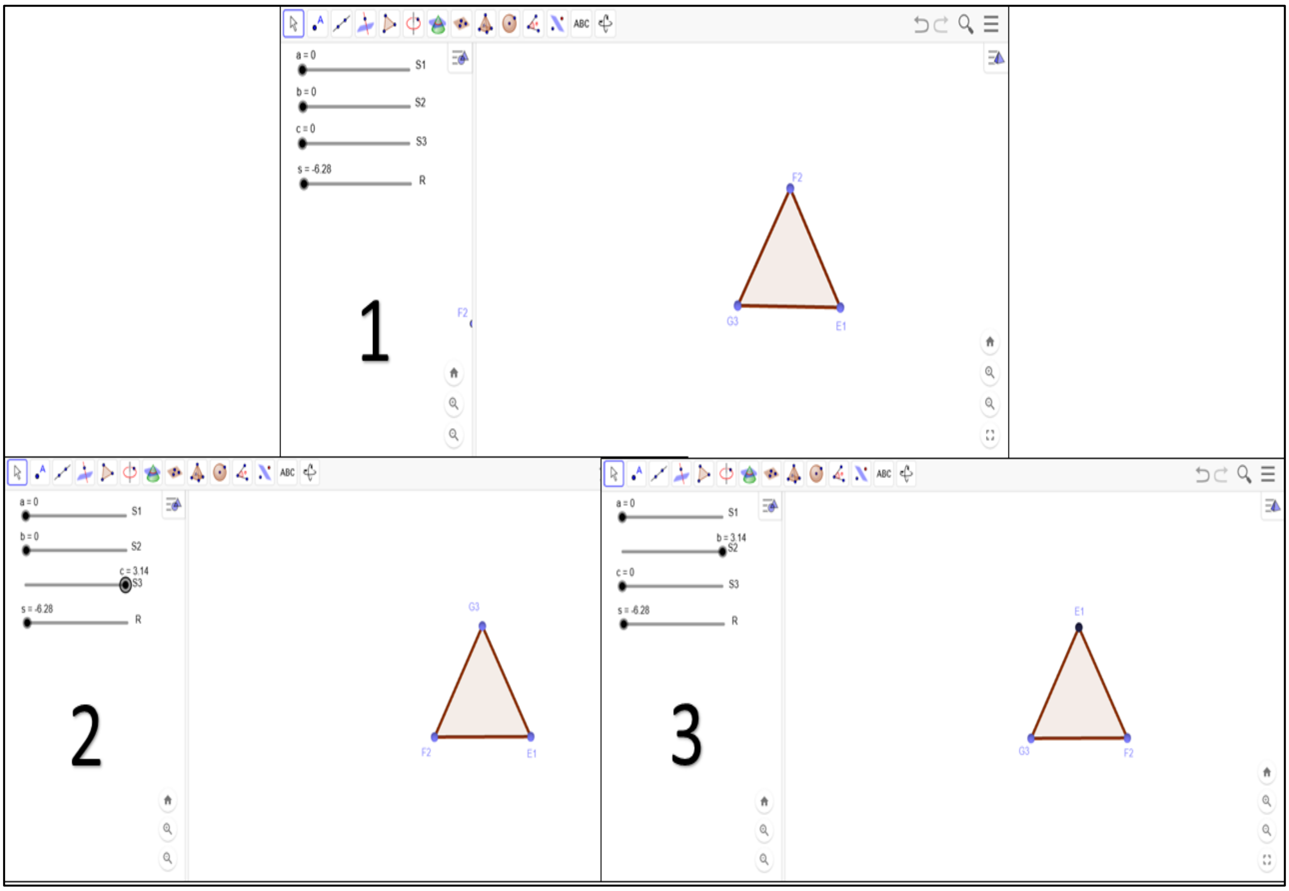

Figure 2. Rotations of the symmetries of the equilateral triangle

The equilateral triangle has a rich set of symmetries, including rotations of 120° and 240° about its center and reflections about its three perpendicular bisectors. These six symmetries form the dihedral group D3. GeoGebra allows the visual construction of these symmetries, facilitating the understanding of the structure of the group. Symmetry is a manifestation of the presence of a symmetry group.

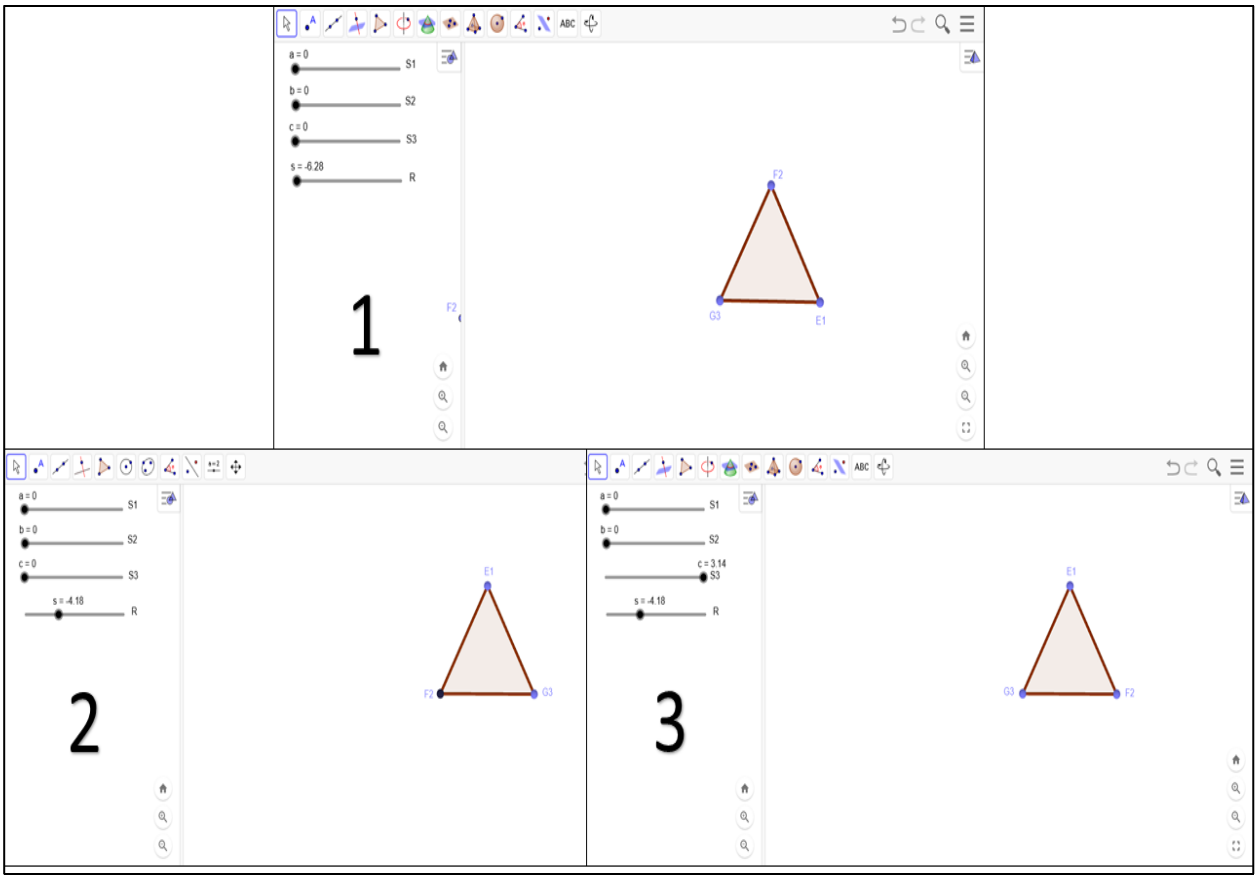

Figure 3. Reflections of the symmetries of the equilateral triangle

An important aspect of group theory is the composition of symmetries. In GeoGebra, it is possible to apply successive symmetries to an object and observe the result. For example, rotating a triangle and then translating it. This visual experimentation helps teachers understand group operations and verify properties such as associativity for a sociocultural practice in the classroom with their students. The teaching proposal with GeoGebra allows a dynamic and interactive approach to the study of symmetry groups. The connection between algebra and geometry becomes more concrete, facilitating the understanding of the concepts involved. This approach can be applied at different levels of education, adapting the activities to the age group and learning objectives. The use of GeoGebra motivates teachers and provides dynamic learning.

Discussion

The use of GeoGebra provided a transformative experience for rural pre-service mathematics teachers in engaging with the concept of symmetry. By allowing dynamic manipulation of geometric figures, the software enabled learners to visualize rotations and reflections in ways that traditional tools could not offer. This aligns with findings by Kovács et al. (2023), who demonstrated that GeoGebra enhances learners' ability to understand geometric conjectures through visual interaction. Participants not only constructed accurate symmetric transformations but also verbalized their reasoning more confidently. Such improvements are crucial for rural teacher training, where instructional resources are often scarce, and technology can compensate for lack of manipulatives (Chehlarova & Gachev, 2023). As learners experimented with the transformation tools, they discovered properties of symmetry such as invariance and composition, thus internalizing abstract group theory. GeoGebra also fostered engagement and motivation, with several participants expressing increased interest in teaching geometry innovatively (Hall & Bastos, 2024). These outcomes support the integration of GeoGebra into pre-service mathematics education, especially in settings where interactivity and feedback are otherwise limited.

Furthermore, the visual and interactive environment of GeoGebra played a central role in supporting cognitive development through constructivist learning. Students transitioned from passive receivers of content to active participants in building knowledge by exploring symmetry relations on their own. As emphasized by Vedrenne-Gutiérrez et al. (2021), digital platforms like GeoGebra provide scaffolding that helps students connect spatial representations with algebraic structures. In our context, learners visualized each symmetry operation, matched it with its corresponding transformation matrix, and identified the resulting figure. This hands-on alignment between visual and symbolic representation helped to demystify abstract mathematical ideas (Hershkovitz et al. 2023). GeoGebra served as a bridge between formal theory and classroom practice, allowing teachers-in-training to better understand how they might replicate this approach with their future students. The iterative feedback loop—visual action followed by reflection—deepened understanding and aligned with current theories of mathematical cognition. These findings further reinforce the importance of technology-enhanced learning in teacher preparation programs. Particularly in rural settings, such tools can offset systemic limitations and democratize access to quality mathematics instruction.

Finally, GeoGebra facilitated a sociocultural learning experience that emphasized collaboration, exploration, and contextual relevance. Teachers were encouraged to connect triangle symmetry with cultural or real-life objects from their local environments, enhancing the contextualization of mathematical learning. This approach aligns with the principles of culturally responsive pedagogy, where content is grounded in learners' backgrounds and everyday experiences. Chehlarova & Gachev (2023) suggested that digital tasks involving symmetries can serve as an entry point for integrating mathematical thinking with artistic and cultural expression. Through peer collaboration and discussion, participants reflected on how to adapt these concepts for younger students using locally available resources. The freedom to experiment, fail, and revise without penalty empowered them to internalize mathematical concepts more confidently. GeoGebra's accessibility was particularly important for these participants, many of whom had never used dynamic geometry software before. The results indicate that with minimal orientation, rural teachers can master and leverage such tools for professional growth. Therefore, the study advocates for institutional support in equipping future educators with digital competencies, starting at the training level.

Research Contribution

This study contributes to the field of mathematics education by demonstrating how dynamic geometry software can effectively enhance teacher training in under-resourced rural environments. Through the integration of GeoGebra, the research offers a practical model for visualizing group-theoretic symmetries "specifically the dihedral group D3" within the context of equilateral triangles. By focusing on pre-service teachers, the study addresses a notable gap in the literature, which often emphasizes student outcomes over teacher preparation. The use of interactive digital tools fosters conceptual clarity, supports reflective learning, and equips future educators with the technological fluency necessary for modern classrooms. This aligns with the findings of Thapa et al. (2022) and Zhang et al. (2025) and Zulnaidi & Zamri. (2017), who noted that GeoGebra enables learners to construct knowledge actively through visual exploration of mathematical structures. Moreover, it reinforces the call by Avsec et al. (2023) and Lepore (2024) and Tran et al. (2017) for leveraging symmetry tasks in digital environments to increase learner engagement and cognitive depth. The research also offers scalable insights into how technology can bridge the pedagogical divide between urban and rural schooling. Ultimately, this contribution underscores the importance of digital equity and content-specific innovation in mathematics teacher education.

Implication

The findings imply that dynamic mathematics software like GeoGebra can significantly enhance pre-service teacher preparation, especially in rural contexts. The integration of interactive visualization tools fosters deeper conceptual understanding and prepares educators to teach geometry more effectively. Educational institutions, particularly those serving under-resourced communities, should consider embedding such tools into their training curricula. GeoGebra also enables culturally relevant instruction by allowing teachers to design lessons grounded in local contexts. Its free availability and low technical requirements make it a scalable solution for equitable education. Moreover, fostering technological fluency among teachers early in their careers can have a long-term impact on instructional quality. The study further contributes to the broader discourse on digital pedagogy and equity in mathematics education. If adopted systematically, tools like GeoGebra could become catalysts for transforming how geometry is taught and learned across diverse educational settings.

Limitations of the Study

This study was conducted with a relatively small sample of 15 pre-service teachers from a single rural teacher education program, which limits the generalizability of the findings. The research design was qualitative and did not incorporate quantitative assessments of learning gains. Additionally, the study relied on self-reported data from interviews, which may introduce bias. The participants' limited prior exposure to technology could also have affected their initial engagement with the software. Furthermore, the study focused exclusively on the topic of equilateral triangle symmetry, potentially limiting its applicability to other geometric concepts. Variability in teaching experience among participants may have influenced the depth of their reflections. Time constraints within the training sessions may have restricted extended exploration of GeoGebra's features. Future studies could expand the sample size, include control groups, and explore longitudinal outcomes.

Suggestions

Based on the results, it is recommended that teacher education institutions incorporate regular training on the use of dynamic mathematics software like GeoGebra. Curriculum designers should consider developing thematic modules focused on specific geometry topics, such as symmetry, rotation, and transformation. Professional development programs should be tailored for rural and low-resource settings, ensuring accessibility and support. Future research could examine the long-term impact of such interventions on classroom teaching effectiveness. There is also a need to explore how GeoGebra can be adapted for primary and secondary school curricula, with culturally contextualized tasks. Collaboration with local education stakeholders may help in creating lesson plans that resonate with community values. Researchers should also investigate how digital tools can support inclusive education, particularly for students with learning differences. Finally, policy-level support is essential to scale up the integration of educational technologies in rural teacher training programs nationwide.

CONCLUSION

This study explored the potential of GeoGebra as a dynamic learning tool to teach the symmetries of the equilateral triangle within the context of rural mathematics teacher training. The findings revealed that participants were able to engage with complex geometric concepts such as rotations and reflections through interactive visualizations, leading to more meaningful understanding. The integration of GeoGebra not only supported individual exploration but also encouraged collaborative learning and the contextualization of content in rural educational environments. As a free and accessible software, GeoGebra provided an equitable digital resource that can bridge the pedagogical gap often experienced in under-resourced schools. Participants demonstrated increased confidence in explaining group structures and applying transformation concepts, suggesting that such digital tools enhance both content mastery and teaching readiness. These results highlight the importance of embedding digital competencies in teacher preparation programs, particularly to equip future educators for the demands of 21st-century classrooms. While limited in scope and duration, the study offers a replicable model for integrating technology into mathematics education through focused, content-specific interventions. Overall, GeoGebra proved to be an effective mediator between abstract theory and practical teaching strategies, affirming its value in transforming geometry instruction for pre-service teachers in rural settings.

AUTHOR CONTRIBUTIONS STATEMENT

All authors contributed meaningfully to the development and completion of this study. The first author was responsible for conceptualizing the research framework, designing the instructional intervention using GeoGebra, and leading the classroom implementation. The second author coordinated the qualitative data collection process, including classroom observations, interviews, and digital activity analysis. The third author contributed to the theoretical alignment of the study, particularly the integration of symmetry and group theory into the research design. All authors were equally involved in coding and analyzing the qualitative data, ensuring the accuracy and reliability of the interpretations. The writing of the manuscript was collaboratively completed, with each author taking responsibility for specific sections including the introduction, literature review, and discussion. Regular team discussions were conducted to ensure conceptual coherence and alignment with international academic standards. All authors reviewed and approved the final version of the manuscript, and they collectively endorse the integrity and originality of this research.

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