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Students' mathematical critical thinking skill: An exploration case study when using argumentation-based learning

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Abstract

Background: Critical thinking skills are essential in addressing the learning challenges faced by students in the 21st century. To foster these skills effectively, it is necessary to adopt specific instructional approaches that enhance students' mathematical critical thinking within the classroom setting.
Aims: This study aims to investigate students' critical thinking abilities in mathematics during the implementation of the Argumentation-Based Learning Process (ABLP) in classroom instruction.
Methods: A qualitative case study design was employed, involving 30 eighth-grade students from an A-accredited school in Indonesia. Data sources included ABLP-based worksheets, audio recordings of student group discussions, and interview transcripts. The collected data were analyzed using a triangulation strategy to ensure accuracy, and peer reviews were conducted to validate the findings. The analysis focused on identifying indicators of critical thinking skills at each stage of the ABLP model.
Results: The results show that students demonstrated various levels of critical thinking throughout the ABLP stages. However, not all students consistently met the predetermined indicators at each stage of the process. Despite these variations, the findings suggest that ABLP can be an effective pedagogical approach to foster students' critical thinking in mathematics.
Conclusion: This study concludes that ABLP offers meaningful opportunities for students to develop critical mathematical thinking and should be considered as a strategy in mathematics instruction.

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INTRODUCTION

Critical thinking is widely recognized as one of the essential competencies in the twenty-first century (Susandi et al., 2019; Lantian et al., 2021; Amanda et al., 2023). This skill plays a vital role in helping students draw conclusions when confronted with problems during classroom learning. In the context of mathematics education, critical thinking allows students to analyze, assess, and make informed decisions when solving problems, leading to accurate and valid conclusions (Monteleone et al., 2018; Susandi, 2021; Umam & Susandi, 2022). Furthermore, mathematical critical thinking encourages students to engage in logical reasoning, reflect deeply on given situations, and determine appropriate solutions (Monteleone & Miller, 2023; Susandi et al., 2018).

Critical thinking in mathematics refers to a cognitive process characterized by rational and reflective reasoning when making decisions based on specific problems (Buphate & Esteban, 2022; Susandi, 2021). It encompasses higher-order thinking and problem-solving capabilities (Huang & Chang, 2022). Moreover, mathematical critical thinking involves the ability to analyze, evaluate, and draw conclusions when responding to given tasks (Iman & Angraini, 2019; Baş & Bolat, 2022). This skill can be observed both during classroom activities through students' behavior and through assessments administered at the end of the learning process (Susandi & Widyawati, 2022; Barker, 2023).

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Based on the 2018 Programme for International Student Assessment (PISA) and the 2019 Trends in International Mathematics and Science Study (TIMSS), Indonesian students consistently perform poorly in mathematics (Kusaeri & Aditomo, 2019; Nasser et al., 2021). These low scores are largely attributed to students' limited experience with mathematical tasks that require critical thinking (Nursyahidah & Albab, 2017; Umam & Susandi, 2022). Moreover, many students lack essential competencies in critical mathematical thinking, particularly in analyzing, evaluating, and drawing conclusions (Susandi et al., 2022). Given these challenges, it is crucial to cultivate critical thinking skills in mathematics through classroom instruction to better equip students in solving mathematical problems effectively (Susandi et al., 2022; Saidin et al., 2024; Yefang et al., 2024).

Implementing learning approaches that foster students' critical mathematical thinking is essential in the educational process (Monteleone et al., 2023; Susandi et al., 2022). One such promising approach is the Argumentation-Based Learning Pedagogy (ABLP), which has been shown to enhance students' ability to think critically in mathematics classrooms (Akbaş, 2021; Baş & Sevim, 2020; Yanti & Susilo, 2024). The ABLP model is particularly valuable because it encourages learners to articulate well-supported claims and make informed decisions based on relevant evidence (Antonio & Prudente, 2021; Seda, 2021). This pedagogy emphasizes the importance of reflective thinking, enabling students to derive more thoughtful conclusions when solving problems presented by the teacher (Türk & Seyhan, 2014; Tüzün et al., 2022). Through ABLP, learners are guided to develop critical competencies such as constructing claims, justifying them with valid reasoning, analyzing and evaluating different arguments, and ultimately drawing sound conclusions aligned with the model's framework (Demircioglu et al., 2022; Özelma & Seyhan, 2022). Each stage of the ABLP process supports students in grounding their decisions with appropriate evidence before reaching a final conclusion (Alt & Kapshuk, 2022; Sarıgöz, 2023).

Based on a review of previous studies, various researchers have explored students' critical thinking skills in mathematics. Demircioglu et al. (2023) found that critical thinking can be enhanced through learning activities that integrate Augmented Reality with argumentation strategies. Similarly, Shanta and Wells (2022) reported that students engaged in STEM-based instruction exhibited strong critical thinking skills, enabling them to make sound decisions in their learning. Furthermore, Susandi et al. (2022) demonstrated that the implementation of the M6 learning model positively influenced students' critical thinking performance during classroom instruction. Although these studies highlight effective strategies for fostering critical thinking, they primarily assess students' cognitive abilities through post-instruction testing. Such assessments often overlook students' critical thinking behaviors as they occur during the learning process itself (Demircioglu et al., 2023; Shanta & Wells, 2022; Susandi et al., 2022). To address this gap, the present study introduces a novel approach by utilizing an argumentation-based learning model to observe and analyze students' mathematical critical thinking in real-time classroom interactions. This approach aims to construct a more comprehensive profile of students' critical thinking abilities in mathematics.

Research focusing on case studies that explore students' mathematical critical thinking through the stages of Argumentation-Based Learning Pedagogy (ABLP) remains limited. Most existing studies on students' critical thinking skills in mathematics employ quantitative approaches with statistical analysis (Demircioglu et al., 2023; Susandi & Widyawati, 2022; Susandi et al., 2022). These studies often assess critical thinking only at the end of the learning process. As a result, there is a lack of insight into how students demonstrate critical thinking during the actual classroom instruction. Therefore, it is essential to investigate the development of critical thinking skills throughout the ABLP stages in order to understand students' behaviors more comprehensively.

This study is significant in addressing that gap. By implementing an argumentation-based learning model, researchers can capture how students engage in critical thinking at each phase of the instructional process. Moreover, identifying students' mathematical critical thinking profiles can

guide the development of more effective instructional models that support the improvement of such skills. Ultimately, understanding students' critical thinking processes in mathematics not only enhances instructional quality but also contributes to informed decisions about their future academic and professional trajectories.

METHOD

Research Design

This study employed a single case study design to investigate students' experiences in Argumentation-Based Learning Pedagogy (ABLP) classrooms, particularly in relation to their critical mathematical thinking skills. The single case study approach was selected for its capacity to provide an in-depth and comprehensive understanding of a specific phenomenon. In this context, it enabled a focused exploration of ABLP through the lens of critical thinking indicators as outlined by Ennis (1996) and Susandi et al. (2022), namely analysis, evaluation, and drawing conclusions. The research was conducted in an offline learning setting, where the researchers designed ABLP-based worksheets to facilitate both individual and collaborative learning activities

Learning Context

The study was carried out in mathematics classrooms that hold an A accreditation status in Indonesia. The participants were eighth-grade junior high school students who had never previously engaged in a similar program, particularly in mathematics lessons. The ABLP (Argumentation-Based Learning Pedagogy) approach required students to make claims, justify those claims with logical reasons, challenge opposing claims, and draw accurate and valid conclusions based on sound reasoning and evidence. To support this process, the researcher developed a dedicated ABLP worksheet, which serves as a guide for students to understand and follow each stage of the ABLP process during classroom instruction.

This collaborative project was designed to address a central challenge, encouraging students to construct a solution based on the given problem. Specifically, the worksheet presents a statistical problem that students must solve. The task is completed both individually and in groups to evaluate the development of students' critical mathematical thinking abilities. A detailed sequence of the ABLP classroom activities is presented in Table 1.

Table 1. ABLP Model Steps and Activities

No	ABLP Model Steps	Critical Thinking Skill Indicators	ABLP Model Activities
1	Grounds	Analyze: students are able to analyze the reasons for the answers that have been given students are able to provide conclusions from the reasons given Evaluate: students are able to provide an assessment of the reasons given Conclude: students are able to provide an assessment of the reasons given	Give reasons why the answer is right/wrong
2	Claim	Analyze: students are able to analyze claims from the reasons given Evaluate: students are able to provide assessments of the claims given Conclude: students are able to provide conclusions from claims based on the reasons given	Provides a correct claim based on the reasons given
3	Warrant	Analyze: students are able to analyze the truth of claims that have been given Evaluate: students are able to provide an assessment of the truth of claims that have been given Conclude: students are able to provide conclusions from justification based on claims that have been given	Provide justification for true claims

No	ABLP Model Steps	Critical Thinking Skill Indicators	ABLP Model Activities
4	Backing	Analyze: students are able to analyze evidence from unacceptable reasons Evaluate: students are able to provide an assessment of the evidence provided Conclude: students are able to provide conclusions from evidence based on unacceptable reasons	Provide evidence when reasons are not accepted
5	Rebuttal	Analyze: students are able to analyze the denial of claims that have been made Evaluate: students are able to provide an assessment of the rebuttal given Conclude: students are able to provide conclusions from the dispute based on claims	Providing a refutation of the claims made
6	Qualifier	Analyze: Drawing conclusions from acceptable claims. students are able to provide assessments of decisions that are given Evaluate: students are able to reject decisions from claims that have been made Conclude: students are able to provide conclusions from decisions based on claims that are given	Draw conclusions from acceptable claims.

Participant

The participants in this study consisted of 30 eighth-grade students from a junior high school in Indonesia that holds an A-level accreditation. These students had no prior experience with the Argumentation-Based Learning Pedagogy (ABLP) model. During the study, they were introduced to and engaged in classroom activities using the ABLP approach. The students were divided into six groups, with each group comprising five members. Within their groups, students worked independently using worksheets provided by the teacher. They were also encouraged to consult their peers if they encountered any difficulties. All participants agreed to follow the established guidelines of the ABLP learning process. The mathematics teacher categorized students based on their performance levels in mathematics. Student achievement was classified into three levels: high (scores above 75), moderate (scores between 66 and 75), and low (scores below 65). A minimum score of 75 was set as the benchmark for mastering mathematical concepts in the school.

Instruments

The data sources in this study consisted of: (1) students' written responses on the ABLP worksheet, (2) audio recordings of group discussions, and (3) audio recordings of student interviews. First, individual written responses were collected from students who completed the ABLP worksheet. This worksheet, designed based on an adapted version of an established framework, guided students through activities aimed at identifying and analyzing problems. Before its implementation, the worksheet was reviewed and validated by three expert validators. Once deemed valid, it was employed in the study to capture students' understanding through questions such as: "Who has the problem?", "What is the problem?", and "Who are the users?". Second, while students engaged in group discussions, their interactions were recorded and later transcribed for analysis. The focus of data collection during these group activities was on the stages of planning, experimenting, evaluating, and decision-making. Third, open-ended interviews were conducted with three students to gather additional insights into their development of critical thinking skills in mathematics.

Data Analysis

To ensure the accuracy of the findings, the three types of data were analyzed through a data triangulation strategy. The researchers identified the stages of the Argumentation-Based Learning Pedagogy (ABLP) and aligned them with the indicators of mathematical critical thinking. All collected data were transcribed and systematically coded. The coding scheme for students' mathematical

critical thinking was adapted from the criteria proposed by Ennis (1996) and Susandi et al. (2022). Each student's response at every stage of the ABLP was carefully examined and assessed based on these criteria. The occurrences of critical thinking indicators were counted at each stage, and the indicators were categorized into two levels to describe the students' critical thinking proficiency. To enhance reliability, the coding process was peer-reviewed. When discrepancies in coding emerged among researchers, discussions were held to reach a consensus on the students' statements.

RESULTS AND DISCUSSION

Results

This study investigates students' mathematical critical thinking skills within the framework of the Argumentation-Based Learning Process (ABLP) in mathematics instruction. The results present a conformity matrix that aligns indicators of critical thinking with each phase of ABLP. Furthermore, the findings are organized around key aspects of argumentation: articulating claims, supporting those claims with logical reasoning, challenging opposing viewpoints, and drawing accurate and evidence-based conclusions.

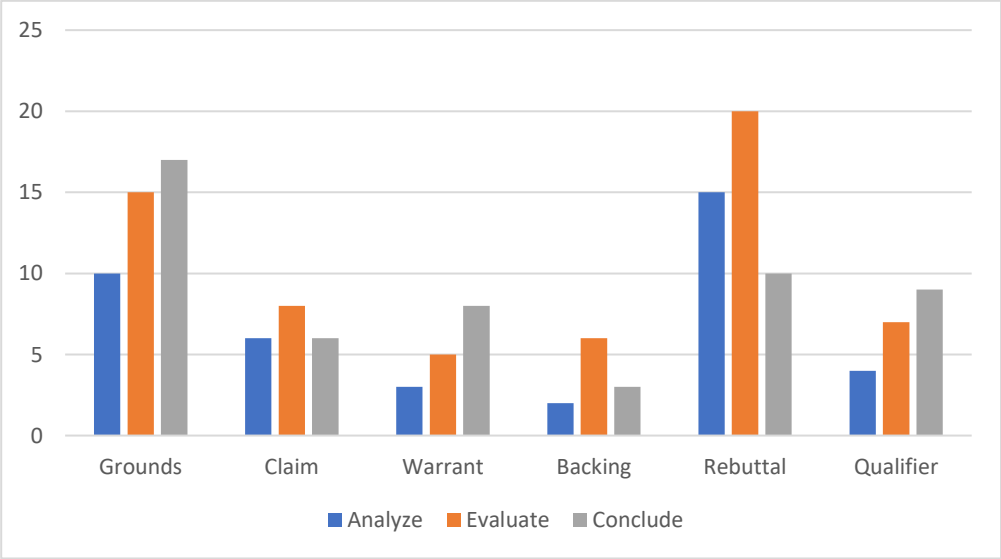
Matching Matrix between Mathematical Critical Thinking Abilities and ABLP Stages

Based on the analysis of student worksheets and classroom discussions, the findings indicate a strong alignment between the indicators of mathematical critical thinking skills and each stage of the Argumentation-Based Learning Pedagogy (ABLP). Students' responses reflecting this relationship are presented in Table 2.

Table 2. Student statements regarding the relationship between indicators of critical thinking abilities in mathematics and ABLP

Indicators of Mathematical Critical Thinking Ability	Steps ABLP					
	Grounds	***Claim***	***Warrant***	***Backing***	***Rebuttal***	***Qualifier***
Analyze	10	6	3	2	15	4
Evaluate	15	8	5	6	20	7
Conclude	17	6	8	3	10	9

Table 2 is a student statement regarding the relationship between indicators of critical thinking ability in mathematics and ABLP which can be presented in the form of a bar diagram. The bar diagram can be seen in Picture 1.



Picture 1. Bar Diagram About the Relationship between Indicators of Mathematical Critical Thinking Ability and ABLP

1. Reasons (Grounds)

The first stage in Argumentation-Based Learning Pedagogy (ABLP) is known as the Grounds phase. In this phase, the teacher presents a mathematical problem that students are expected to solve collaboratively in groups. Students are required to justify whether the given solution is correct or incorrect. This activity is designed to help students develop well-supported claims that can lead to valid conclusions. The worksheet includes multiple possible answers to the problem, each requiring students to provide logical reasoning.

Based on Picture 1, in Step I, only 10 student responses aligned with the indicators of analytical ability. Additionally, only 15 students demonstrated achievement of the evaluation skill indicators, and just 17 students were able to draw appropriate conclusions. These observational findings required further clarification from the students involved. Therefore, an interview was conducted with one of the students. A portion of the interview transcript is presented below:

Q: Hey, what is the correct answer to this question?
S1: answers from the first student and the third student
Q: why answer that?
S1: the answers of the first student and the second student are correct because based on what is known in the question Adi bought at Shop A and at Shop B so that the system of equations is obtained
Q: What about the second student's answer?
SI: Wrong, sir.
Q: whyMathematics?
S1: because the system of equations formed does not match what is known in the problem

2. Provide the correct claim (Claim)

The next step in Argumentation-Based Learning Pedagogy (ABLP) is the Claim stage. In this phase, students are encouraged to formulate accurate claims based on the reasons they previously expressed. This process is intended to guide students toward identifying a single, well-supported claim derived from their earlier arguments.

Referring to Figure 1, it was observed that in Step II, only six student responses aligned with the indicators of analytical ability. Additionally, just eight students demonstrated the ability to meet the evaluation criteria, and only six students successfully fulfilled the indicators related to drawing conclusions. These observational findings warranted further clarification through direct confirmation. Therefore, an interview was conducted with one of the students. The following is an excerpt from the interview:

Q: So what decision can be made based on the reasons given earlier?
S1: In my opinion, yes, sir, if the first student and third student give the right answer while the second student gives the wrong answer
Q: Are you sure about the decision you made?
S1: very sure, sir, because I decided based on the data known in the matter, sir

3. Providing Justification for True Claims (Warrant)

The next phase in the Argumentation-Based Learning Pedagogy (ABLP) is the Warrant stage. At this point, students are expected to present evidence that supports the validity of claims they believe to be true. This process is essential for helping students develop confidence in their assertions, as it requires them to verify their claims using reliable sources or data searches.

According to the data presented in Figure 1, during Step III, only three student responses aligned with the indicators of analytical ability. Additionally, only five students demonstrated competencies associated with evaluative thinking, while eight students showed the ability to draw appropriate conclusions. These observational findings prompted further validation through student

interviews. As such, one student was selected for an interview, and the following is an excerpt from that conversation:

Q: What evidence makes the decision taken correct?

S1: Well, sir, the question is that Adi is required to bring scout equipment. At shop "A" Adi bought 4 ropes and 2 sticks for IDR 40,000.00 and bought 2 scarves and 3 ropes for IDR 30,000.00. Because Adi still needed to buy ropes, sticks and scarves while the supplies at shop "A" were running out, Adi moved to shop "B" to buy the scout equipment. At shop "B" Adi bought 2 ropes and 1 stick for IDR 20,000.00 and bought 2 sticks and 4 scarves for IDR 50,000.00. So, based on this data, I am sure that the answers from the first and third students are correct, Sir.

Q: Then why is Adek sure that the second student's answer is wrong?

S1: I see from what is known in the question, Sir.. The second student's answer, the numbers do not match what is known in the question.

4. Providing Evidence when the Reason is not accepted (Backing)

The subsequent phase in ABLP is Backing. In this stage, students are encouraged to provide additional or alternative evidence to support claims they believe to be valid. This step is necessary when the initial evidence presented by other students is not accepted by their peers. Therefore, students must offer further justification to strengthen the credibility of their claims and gain peer acceptance.

As shown in Figure 1, only two student responses in Step IV aligned with the indicators of analytical thinking. Additionally, only six students met the criteria for evaluation skills, and just three students demonstrated the ability to draw conclusions. These observational findings required further clarification. To validate the data, an interview was conducted with one of the students. An excerpt from the student interview is presented below:

Q: Is there any other evidence of the decision taken?

S1: of course there is sir...

Q: What is that, Deck?

S1: I immediately looked at the numbers in the questions and then looked at the answer choices from the three students. Then I immediately guessed that the students whose answers were correct were the first and third students.

5. Provide a refutation of the claims made (Rebuttal)

The first phase of the Argumentation-Based Learning Pedagogy (ABLP) is Rebuttal. In this stage, students are encouraged to present rebuttals to challenge or refine existing claims by drawing on both initial and supplementary evidence. This process is designed to cultivate students' ability to critically evaluate arguments and reinforce their confidence in claims that are supported by valid and reliable evidence.

As illustrated in Figure 1, during Step V, only 15 student responses aligned with the indicators of analytical thinking skills. Furthermore, just 20 students demonstrated the ability to meet the evaluation criteria, and only 10 students were able to fulfill the indicators related to drawing conclusions. These observational findings require validation through direct student input. Consequently, an interview was conducted with one of the students. The following excerpt presents a portion of that interview:

Q: Do you see answers that are different from your answers in class?

S1: Yes sir

Q: So what is your attitude?

S1: I immediately denied sir that the answer was wrong.

Q: Which answer is different from Adek's?

S1: the second student's answer is considered by my friend to be correct. Even though I have proven that student two's answer is wrong

6. Make Conclusions about Acceptable Claims (Qualifier)

The final stage of the Argumentation-Based Learning Pedagogy (ABLP) is the Qualifier. At this point, students are encouraged to validate and justify claims that can be accepted collectively by the class. This process allows students to reach a consensus on conclusions that are deemed accurate and valid throughout the learning process.

Referring to Figure 1, in Step VI, only four student responses aligned with the indicators of analytical ability. Additionally, only seven students met the criteria for evaluative ability, and just nine demonstrated the ability to draw valid conclusions. These observational findings required further clarification. Therefore, an interview was conducted with one of the students to gain deeper insight. The following is an excerpt from the interview:

Q: From the answers you wrote on the answer sheet, what conclusions can you draw?

S1: In my opinion, from the questions I have worked on, after I prove it, it can be concluded that the answers of the first and third students are considered correct, while the answers of the second student are considered wrong

Discussion

This study highlights the effectiveness of Argumentation-Based Learning Pedagogy (ABLP) in enhancing students' critical mathematical thinking. Each phase of ABLP provides opportunities to observe how students develop and demonstrate their critical thinking skills. These results are consistent with the findings of Fettahlıoğlu and Aydoğdu (2020), who noted that ABLP significantly fosters critical mathematical thinking, especially during the process of making and challenging claims. Such interactive learning environments encourage learners to critically examine responses, justify their reasoning, and refine their arguments, thereby strengthening their overall critical thinking abilities (Susandi et al., 2019). Collaborative group work also supports this development, as students exchange perspectives, provide justifications, and articulate claims they believe to be valid (Marthaliakirana et al., 2022; Taiwo & Ige, 2023). The success of ABLP aligns well with constructivist learning theories, as it emphasizes evidence-based reasoning and active student participation. Nevertheless, its effectiveness may be influenced by contextual factors, including students' experience with structured argumentation, the role of the teacher in facilitating discourse, and cultural norms regarding critical questioning and dialogue (Tuysuz & Tuzun, 2020; Suparman & Juandi, 2023).

The objective of the ABLP (Argumentation-Based Learning Pedagogy) class is to enable students to make informed decisions about determining the price of one rope and one stick based on a given linear equation. Through the structured steps of ABLP, it becomes evident that students develop their critical mathematical thinking skills, particularly in the process of decision-making. ABLP is grounded in constructivist learning theory, as it encourages active student interaction at each stage. Students are required to express claims supported by evidence, engage in discussions, and critique one another's reasoning. Furthermore, within the ABLP framework, students construct their own solutions by applying their understanding of mathematical concepts, leading to accurate and valid conclusions. This approach supports the enhancement of students' critical thinking in mathematics (Susandi et al., 2022; Tamah et al., 2023; Mastnak et al., 2023). For instance, student A1, who initially demonstrated low levels of critical thinking, showed significant improvement after participating in ABLP activities. This suggests that collaborative engagement—particularly through challenging and refining claims—can effectively strengthen students' mathematical critical thinking skills (Susandi et al., 2019; Fatmanissa et al., 2023).

Students are encouraged to present evidence to support their claims during the stages of Argumentation-Based Learning Pedagogy (ABLP), which contributes to the development of critical thinking skills in mathematics (Reuter, 2023). As a result, they construct arguments using logical reasoning drawn from multiple sources, including group discussions. These activities ultimately require students to engage in mathematical critical thinking when reaching conclusions and identifying correct answers (Demircioglu et al., 2023; Hasan et al., 2023). This study highlights that the ABLP framework can effectively reveal specific indicators of students' mathematical critical thinking abilities.

Mathematical critical thinking skills can be nurtured through each phase of the Argumentation-Based Learning Pedagogy (ABLP). During the grounded phase, students begin to demonstrate critical thinking, particularly in making conclusions. This is due to their engagement with deep conceptual understanding, which enables them to solve problems effectively (Petrulytė et al., 2020; Jacinto & Carreira, 2023; Anupan & Chimmalee, 2024). Moreover, in this phase, students are encouraged to present reasoning and justification when solving mathematical problems, thereby promoting the development of logical thinking (Colonnese & Casto, 2023; Nelisiwea & Yu, 2024). In the claim phase, students' critical thinking also progresses, especially in terms of evaluation. This is because they are expected to critically analyze mathematical problems and assess their conclusions based on supporting evidence and logical reasoning (Kaya & Kesan, 2023; Ahliodžić et al. 2024). Additionally, this phase provides opportunities for learners to connect theoretical mathematical concepts with practical problem-solving, leading to the construction of coherent and logical claims (Brewster & Miller, 2023; Beck et al., 2024).

In the Warrant stage, students' critical thinking skills are fostered during classroom learning, particularly in terms of drawing conclusions. This stage allows students to construct clear and specific claims based on the given mathematical problems (Demiray et al, 2023; Rushton et al., 2024). Additionally, they are encouraged to provide thorough mathematical justifications and apply relevant concepts to support their claims (Caviedes et al., 2023; Uzun, 2024). Meanwhile, in the Backing stage, critical thinking is further developed, especially in the aspect of evaluation. At this point, students are expected to critically assess the reasons or evidence provided (Rizos & Gkrekas, 2023; Negara et al, 2024). Furthermore, they must apply logical reasoning to effectively connect their claims with supporting mathematical evidence (Wang, 2024).

In the Rebuttal phase, students' critical thinking skills are fostered particularly in relation to the evaluation indicator. During this stage, they are encouraged to critically assess mathematical arguments, enabling them to identify flaws or weaknesses in the claims presented (Meena & Lakshmi, 2023; Rushton et al., 2024). Moreover, students are given opportunities to construct rebuttals by presenting mathematical evidence and alternative reasoning. This process helps them evaluate the validity of mathematical claims from multiple perspectives and encourages deeper problem-solving (Rizos & Gkrekas, 2023; Uzun, 2024). Subsequently, in the Qualification stage, critical thinking is further developed, particularly through the conclusion indicator. Students are guided to draw logical conclusions or generalizations based on mathematical arguments presented. This step not only helps validate the claims but also encourages learners to reflect on their reasoning and critically examine the soundness of their conclusions, thereby enhancing the overall quality of mathematical argumentation (Seifert et al., 2022; Rushton et al., 2024; Solar et al., 2022; Uzun, 2024).

Moreover, this study contributes significantly by emphasizing that assessing students' critical thinking skills in mathematics must align with performance indicators such as the ability to analyze, evaluate, and draw conclusions. Specifically, students are expected to justify claims with valid reasoning and evidence, enabling them to develop appropriate solutions to given problems. The implementation of the Argumentation-Based Learning Pedagogy (ABLP) also creates opportunities for students to engage in discussions, express their claims, and critically challenge one another's

ideas. Through this interactive process, learners are encouraged to construct more refined solutions. In addition, the study encourages students to revisit and refine accurate claims collaboratively, fostering deeper development of their mathematical critical thinking abilities. This research differs from prior studies, which primarily examined the relationship between ABLP and students' critical thinking skills by administering only post-tests (Demircioglu et al., 2023; Sarıgöz, 2023).

This study has several limitations. It primarily examines critical thinking skills within the context of Argumentation-Based Learning Pedagogy (ABLP) in mathematics, while other influential factors" such as emotional intelligence, learning motivation, and individual learning styles" were not explored. Additionally, the research employed a single case study involving students from one school, which may limit the generalizability of the findings to broader student populations with diverse backgrounds. The study was also conducted within a relatively short timeframe, preventing a long-term assessment of students' critical thinking development. Further research is needed to evaluate the long-term effectiveness of ABLP in enhancing mathematical critical thinking skills

CONCLUSION

This study demonstrates that Argument-Based Learning Pedagogy (ABLP) effectively fosters pupils' critical mathematical thinking by engaging them in structured argumentation and collaborative problem-solving. Within the case study conducted, indicators of critical mathematical thinking were apparent through the sequential stages of ABLP. The findings reveal that ABLP enhances pupils' ability to analyse, evaluate, and draw conclusions from mathematical arguments. In particular, pupils were seen refuting claims, justifying solutions with sound mathematical reasoning, and collaboratively verifying their conclusions. ABLP enables pupils to work in groups to examine and assess peer assertions in order to determine the most accurate and logically valid responses. Additionally, pupils are involved in challenging the claims presented. This process allows them to practise the core components of critical thinking, including analysis, evaluation, and drawing conclusions. Once they have agreed upon the correct claims for problem-solving, they re-examine whether the results align with the problem posed. After confirming this, pupils collectively determine the most appropriate answer. These findings suggest that critical thinking skills in mathematics should be assessed throughout the learning process to observe how such skills develop over time. For teachers, ABLP offers a structured framework for teaching critical thinking within mathematics lessons. Through its stages, the pedagogy supports the development of 21st-century competencies in mathematical education. Policy-makers are encouraged to support the integration of ABLP into the school curriculum, as the approach has been shown to cultivate positive engagement in critical mathematical thinking. However, this research is limited by its sample size. Further studies involving a larger group of participants are recommended to ensure consistency and broader applicability of the results.

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AUTHOR CONTRIBUTIONS STATEMENT

ADS was responsible for the conceptualisation, design of the study, and manuscript preparation. SD contributed to data collection, transcription, and analysis. Both authors reviewed and approved the final version of the manuscript.

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