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Exploring students' mathematical reasoning in solving HOTs problems based on thinking styles

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Abstract

Background: The advancement of technology has facilitated rapid access to information, yet it poses challenges in discerning accurate information. In this context, critical thinking becomes essential for analyzing and evaluating information. Within mathematics education, exploring students' reasoning processes and their alignment with thinking styles is crucial for enhancing problem-solving skills, especially in addressing Higher Order Thinking Skills (HOTS) problems.
Aims: This study aims to describe students' mathematical reasoning in solving HOTS problems on the topic of systems of three-variable linear equations, focusing on two distinct thinking styles: Abstract Random and Concrete Sequential.
Methods: This qualitative descriptive study was conducted at Cokroaminoto Palopo University with 36 Mathematics Education students. Two subjects, representing each thinking style, were purposively selected based on a thinking style test. Data collection involved mathematical reasoning tests, interviews, and observations, with the researcher serving as the primary instrument.
Results: The findings indicate that both Abstract Random and Concrete Sequential subjects demonstrated reasoning abilities that align with all six indicators of mathematical reasoning. Notably, the Abstract Random subject approached problems through hypothesis formation and fractional equations, while the Concrete Sequential subject systematically assigned values and developed mathematical models. Both subjects re-checked their solutions to ensure accuracy.
Conclusion: This study concludes that students with both Abstract Random and Concrete Sequential thinking styles exhibit effective mathematical reasoning when solving HOTS problems. These results highlight the importance of tailoring instructional strategies to accommodate diverse thinking styles to enhance students' reasoning abilities in mathematics education.

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INTRODUCTION

Mathematical reasoning is increasingly recognized as a cornerstone of mathematics education, essential for equipping students with 21st-century skills such as critical thinking, problem-solving, and mathematical literacy. Higher Order Thinking Skills (HOTS), encompassing advanced cognitive processes such as analysis, synthesis, and evaluation, are critical for addressing unfamiliar challenges and making informed decisions (Dubas & Toledo, 2016; Nguyen & Nguyen, 2022). Effective teaching practices that emphasize HOTS have been shown to enhance students' engagement, critical thinking, and problem-solving abilities, underscoring the importance of teachers' understanding and

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application of HOTS-oriented strategies in classrooms (Rasyid et al., 2021; Retnawati et al., 2018). Moreover, research demonstrates that engaging students in tasks that require higher-order thinking enables them to transfer knowledge across contexts and apply it to real-world situations, further supporting the development of essential 21st-century competencies (Dubas & Toledo, 2016; Tajudin & Chinnappan, 2016). This emphasis on HOTS is reflected in curriculum reforms globally, including Indonesia's 2013 curriculum, which prioritizes the cultivation of critical and creative thinking skills among students (Utami et al., 2019). By integrating HOTS into mathematics education, educators aim to strengthen students' mathematical reasoning, better preparing them to navigate the complexities of an interconnected and dynamic world.

Building on the significance of HOTS, mathematical reasoning is integral to equipping students with the skills necessary to tackle higher-order mathematical problems. It enables them to analyze complex scenarios, construct coherent arguments, and draw logical conclusions. For instance, solving systems of three-variable linear equations requires students to develop mathematical models, identify patterns, and employ systematic problem-solving strategies (Aiym et al., 2022; Huang & Yidi, 2022). The role of argumentation is particularly noteworthy in this context, as it encourages students to communicate their thought processes effectively and collaborate on finding solutions to challenging problems (Castro et al., 2021; Kartika et al., 2024). Jeannotte & Kieran (2017) propose a framework for mathematical reasoning that emphasizes the integration of structural and procedural components, aligning with findings by Hadi & Zaidah (2020), who identified difficulties students face in representing and analyzing HOTS-related tasks. These perspectives highlight the importance of fostering mathematical reasoning to empower students in addressing the complexities of HOTS-based challenges. Furthermore, understanding individual cognitive differences among students, such as their thinking styles, can provide valuable insights into how reasoning strategies are developed and applied.

Expanding on the relationship between mathematical reasoning and cognitive differences, thinking styles offer a deeper perspective on how students approach and adapt their reasoning strategies. Defined as cognitive preferences that influence how individuals process information, thinking styles significantly impact mathematical reasoning and problem-solving strategies. Abstract Random thinkers often rely on holistic and intuitive approaches, while Concrete Sequential thinkers prefer structured, step-by-step methodologies. These distinctions are particularly important in understanding how students tackle HOTS problems. Research highlights that thinking styles affect mathematical critical thinking abilities, as shown by (Susilo, 2022), who found that kinesthetic learners excel in critical thinking tasks, and Djadir et al. (2018), who noted that Abstract Random thinkers tend to achieve higher success in mathematical learning compared to their Concrete Sequential counterparts. Further evidence underscores the importance of aligning teaching practices with cognitive preferences; Huincahue et al. (2021) demonstrated that analytic thinkers often excel in mathematics, while Fazrianti et al. (2022) emphasized that mathematical analogical reasoning abilities are positively influenced by thinking styles. By understanding these cognitive differences, educators can develop tailored strategies that not only enhance students' mathematical reasoning but also foster critical thinking and problem-solving skills.

Building on the role of thinking styles in shaping mathematical reasoning, previous research has extensively studied mathematical reasoning as a critical cognitive skill supporting HOTS-based learning. These studies demonstrate the effectiveness of HOTS approaches in enhancing students' critical thinking (Lusiana et al., 2024), mathematical literacy (Ismail et al., 2024; Luzyawati et al., 2025), and problem-solving skills (Nindiasari et al., 2024). Purnomo et al. (2024) identified key stages in solving HOTS problems in Differential Calculus, while Juniati and Budayasa (2024) explored the influence of learning styles and working memory on prospective mathematics teachers' performance in solving HOTS problems. Additionally, instructional innovations like flipped classrooms (Harun et al., 2024), GeoGebra (Abd Rahman et al., 2024), and augmented reality (Cai et al., 2019; Demitriadou et al., 2019) have enhanced conceptual understanding and higher-order thinking skills. However, these studies primarily focused on instructional strategies or specific mathematical content without delving into the relationship between thinking styles and the reasoning process. Moreover, culturally-oriented research, such as Hariyadi's (2021) validation of HOTS instruments, provided valuable insights but did not examine how individual thinking styles influence the application of these instruments. While previous studies have contributed significantly to understanding HOTS-based learning, the integration of thinking styles into the exploration of mathematical reasoning processes remains underexplored. This study bridges this gap by investigating how distinct thinking styles, specifically Abstract Random and Concrete Sequential, shape students' mathematical reasoning in solving HOTS problems. The findings aim to offer new insights for cognitively-oriented instructional designs and enrich the existing literature on mathematical reasoning and thinking styles.

METHOD

Research Design

This study employs a qualitative descriptive research design to explore students' mathematical reasoning abilities when solving higher-order thinking skills (HOTS) problems. The research focuses on understanding how students develop and adapt their reasoning strategies, with particular attention to their thinking styles, specifically Abstract Random and Concrete Sequential. This approach seeks to provide a detailed and holistic description of the behaviors, perceptions, motivations, and cognitive processes involved in tackling complex mathematical tasks. Data is collected and analyzed within a natural context, using tools such as observations, interviews, and problem-based assessments to ensure a comprehensive understanding of the phenomena. By examining these cognitive differences, the study aims to uncover how thinking styles influence mathematical reasoning, offering valuable insights for designing instructional strategies that address diverse cognitive preferences.

Participant

This research was carried out at Cokroaminoto Palopo University, located at Jl. Latamacelling No. 19, Palopo City, South Sulawesi Province. The study focused on students enrolled in the Mathematics Education Program within the Faculty of Teacher Training and Education. Out of a total of 36 students in the program, two were purposefully selected as research subjects to provide in-depth insights into their mathematical reasoning abilities. The selection process was designed to represent contrasting thinking styles: one student with an Abstract Random thinking style, characterized by holistic and intuitive problem-solving approaches, and another with a Concrete Sequential thinking style, known for structured and step-by-step reasoning. The selection aimed to explore how these distinct cognitive preferences influence the strategies students use to tackle higher-order thinking skills (HOTS) problems. To ensure rigor and relevance, the selection process followed specific criteria, which are illustrated in the flowchart below, offering a clear overview of how the participants were identified and chosen for the study.

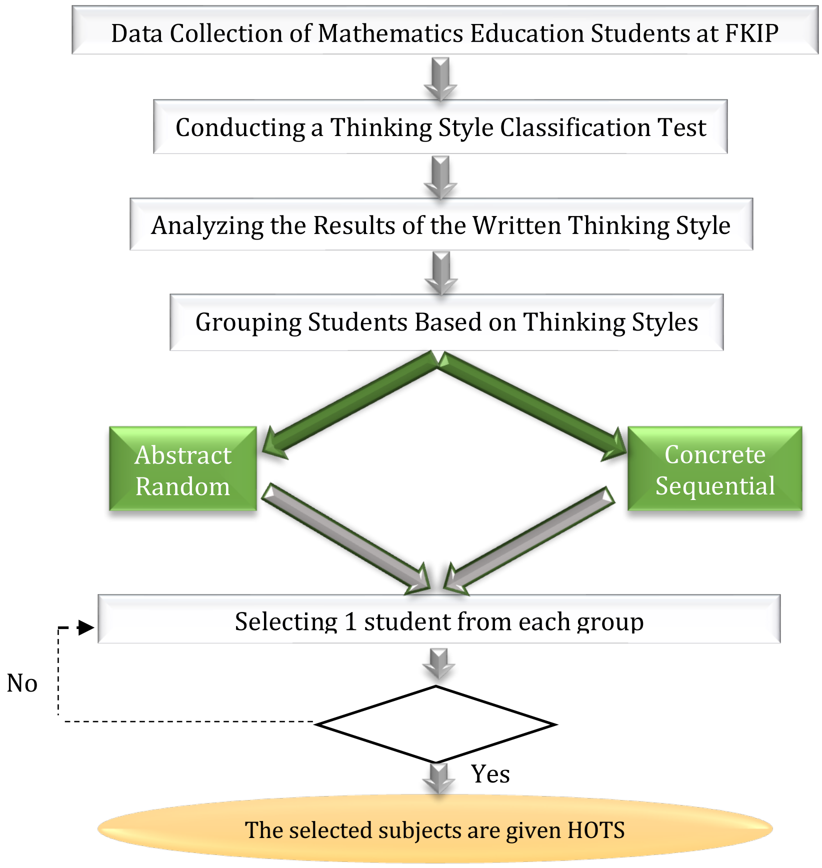

Figure 1. Flowchart of Subject Selection in Research

Instrument

The primary instrument in this study is the researcher. As the main instrument, the researcher is responsible for planning, conducting, collecting data, analyzing, interpreting the findings, and reporting the results of the study. Acting as a facilitator, the researcher enables the exploration of intriguing and unique information, including unexpected or unplanned findings, as suggested by Sugiyono (2012). To complement this role, supporting instruments were employed, including a thinking style test, a mathematical reasoning test, and an interview guide. The thinking style test aims to identify students' cognitive preferences, specifically focusing on Abstract Random and Concrete Sequential thinking styles. This test consists of 15 items, each presenting four statements to evaluate the students' dominant thinking style. The mathematical reasoning test is designed to diagnose students' reasoning abilities when solving Higher-Order Thinking Skills (HOTS) problems. The HOTS questions assess students' ability to analyze, synthesize, and evaluate mathematical scenarios requiring advanced cognitive processes. One example of a HOTS problem used in the study is as follows:

"Three painters, Joni, Deni, and Ari, customarily work together. They can paint the exterior of a house in 10 hours. Deni and Ari have previously worked together to paint a similar house in 15 hours. One day, the three painters worked together for 4 hours on a similar house. However, Ari had to leave unexpectedly due to an urgent matter. Joni and Deni required an additional 8 hours to complete the painting. Determine the time required for each painter to finish the painting individually!"

Data Collection and Analysis

The collected data, including recorded data, was subsequently transformed into interview transcripts. The transcripts and the results of the mathematical reasoning tests related to HOTS (Higher-Order Thinking Skills) problems were analyzed through the following steps: (1) Reviewing all available data from various sources, including interviews, observations documented in field notes, and the results of HOTS reasoning tests, (2) Data reduction, which involves selecting, focusing, abstracting, and transforming raw data, (3) Data presentation, including classification and identification of data, (4) Creating codes to facilitate the representation of students' reasoning data in solving HOTS problems based on their thinking styles, (5) Presenting interview data from the results of students' reasoning tests in solving HOTS problems, and (6) Interpreting the data and drawing conclusions from the collected data, followed by verifying the conclusions.

RESULTS AND DISCUSSION

Results

The selection of research subjects was based on the results of a thinking style questionnaire, which categorized participants into two thinking styles: abstract random and concrete sequential. The following table presents the results of the thinking style test administered to 50 students from two cohorts: the 2017 cohort and the 2018 cohort.

Tabel 1. The Results of Thinking Style Categorization

Thinking Style Categories	Number of Students
abstract random	19
concrete sequential	17
Total Number of Students	**36**

Source: Processed primary data (2021)

Based on Table 1, it was found that out of 36 students, 19 students exhibited an abstract random thinking style, while 17 students demonstrated a concrete sequential thinking style. The selection of subjects was based on the following criteria: (1) willingness to participate as research subjects; (2) each having either an abstract random or concrete sequential thinking style; (3) having received instruction in linear algebra; (4) possessing good communication skills and the ability to express their thoughts and feelings verbally.

Data Presentation of Research Results for Subjects with Abstract Random Thinking Style (AA)

The following are the responses from subjects with an Abstract Random (AA) thinking style for the mathematical reasoning test on Higher Order Thinking Skills (HOTS) problems.

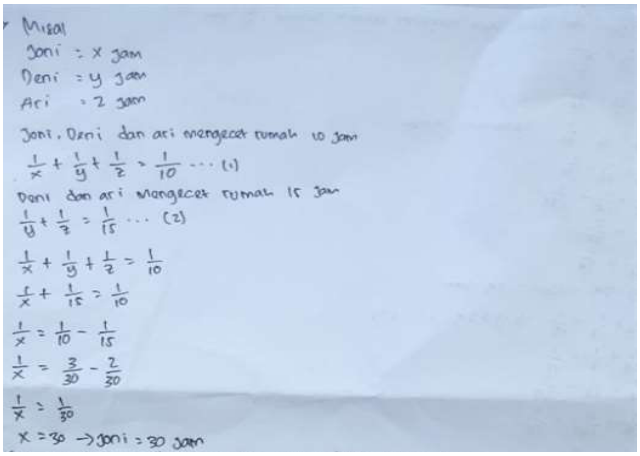

Figure 2. Responses to the Mathematical Reasoning Test for Subjects with Abstract Random Thinking Style

In answering the reasoning test related to the HOTS problem, as shown in the image above, the subject did not write down the given and asked parts of the question. First, the subject assigned variables $x, y, and\ z$ to represent Joni, Deni, and Ari, respectively. Then, the subject formulated two equations based on the problem statement. The first equation was $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ and the second equation was $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$. The subject then substituted the second equation into the first equation to solve for x, where they replaced $\frac{1}{y}+\frac{1}{z}$ with $\frac{1}{15}$ obtaining $x = 30$. Afterward, the subject carefully re-read the question and revised the statement in the section that mentioned "the three painters worked together to paint a similar house for 4 hours.

Afterward, Ari left due to an urgent matter, and Joni and Deni required an additional 8 hours to complete the painting. The subject noted that the three painters together could finish the painting in 10 hours, meaning they could paint $\frac{1}{10}$ of the house per hour. The subject calculated that if all three worked for 4 hours, the portion of the house painted would be $4 * \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$. The subject then subtracted this portion from the whole, resulting in $1 - \frac{1}{2} = \frac{3}{5}$, which represents the remaining work that was completed by Deni and Ari in the next 8 hours. Next, the subject created a ratio: $\frac{3}{5}$ of the work = 8 hours. From this, the subject calculated the time required for Deni and Ari to complete the entire job: $8 * \frac{10}{6} = \frac{40}{3}\ jam$. This is shown in the image below.

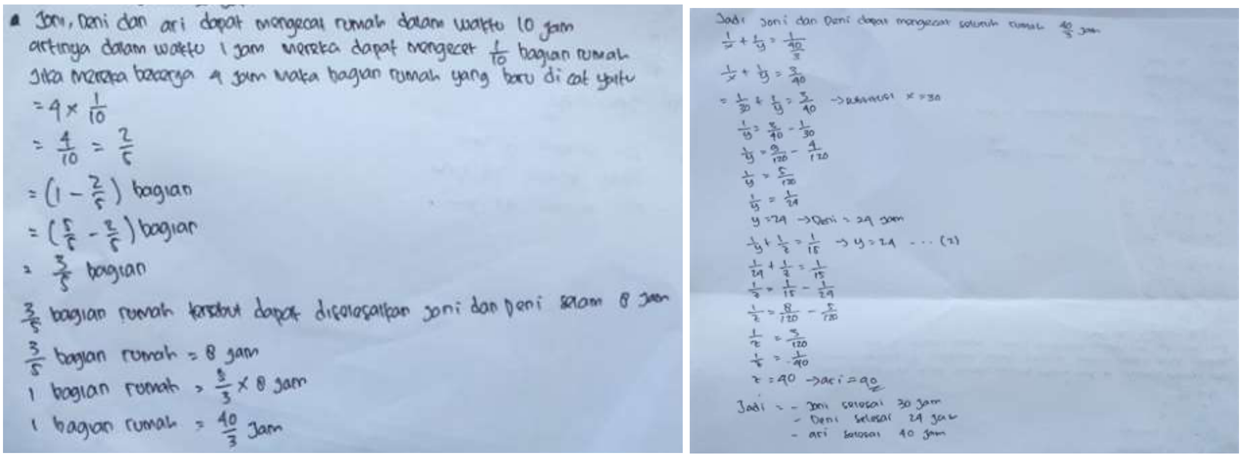

Figure 3. Mathematical Reasoning Test Responses for Subjects with Abstract Random Thinking Style

After determining the time, the subject formulated the equation: $\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{40}{3}} \rightarrow \frac{1}{x}+\frac{1}{y}=\frac{3}{40}$. The subject then substituted $x = 30$ into the equation, resulting in: $\frac{1}{30}+\frac{1}{y}=\frac{3}{40}$. Solving for y, the subject found $y = 24$ hours. The subject then substituted $y = 24$ into the second equation, resulting in: $z = 40$ jam. After obtaining the values for all three variables, the subject concluded that the time required to complete the painting was 30 hours for Joni, 24 hours for Deni, and 40 hours for Ari. Based on the results of the mathematical reasoning test and interviews with the subject having an abstract random thinking style, the following findings were obtained:

Tabel 2. Findings Based on the Results of the Mathematical Reasoning Test and Interviews with Subjects Abstract Random Thinking Style

Valid data	***Code***
Indicator for Presenting Mathematical Statements Verbally, in Writing, and through Diagrams and Figures	
The subject explains the presentation of the problem using models, diagrams, factors, and the relationships within the problem by expressing the equation in fractional form. This approach was based on the subject's reasoning that, since the problem involves time (in hours), it would be	DV-AA-02

Valid data	***Code***
easier to represent the relationships as fractions or parts of a whole. The subject then created two equations to model the problem $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ and $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$	
Indicator for Making Hypotheses or Proposing Assumptions	
The subject begins by assigning variables to represent the painters: Joni, Deni, and Ari, using x, y and z, respectively. The subject then constructs two equations based on the information provided in the problem. The first equation is $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ This equation represents the combined work rate of the three painters (Joni, Deni, and Ari), where they can collectively paint the house in 10 hours. The second equation is: $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$. This equation represents the combined work rate of Deni and Ari, who can paint the house together in 15 hours. With these two equations, the subject is tasked with determining the individual time each painter (Joni, Deni, and Ari) would take to paint the house if working alone. To solve this, the subject uses the **substitution method**.	DV-AA-04
Indicator for Performing Mathematical Manipulation	
The subject wrote and explained the steps in solving the problem by first stating equation 1 as $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ and equation 2 as $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$ then performed substitution to obtain the value of $x=30$. The subject further explained that instead of presenting the equation directly, they made an estimation method and derived the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{40}{3}}$, The subject then continued with substitution and obtained the values of $y=24$ and $z=40$	DV-AA-06
Indicator for Constructing Proof and Providing Justification for the Correctness of the Solution	
The subject wrote and explained that they were unsure how to form the equation for the final statement but instead made an estimation based on the problem statement, resulting in the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{40}{3}}$, The subject then performed substitution with $x=30$ obtaining $y=24$ and $z=40$. Thus, the final values for x, y and z were determined.	DV-AA-08
Indicator for Drawing Conclusions from Statements	
The subject wrote and explained that after obtaining the values for x, y *and* z they returned to the initial assumptions, where x represents the number of hours Joni worked, which is 30 hours, y represents the number of hours Deni worked, which is 24 hours, and z represents the number of hours Ari worked, which is 40 hours.	DV-AA-10
Indicator for Verifying the Validity of an Argument	
The subject stated that they had doubts, but they completed the task according to what they knew. They mentioned that, if their answer turned out to be incorrect, at least they had challenged themselves in attempting to solve the problem.	DV-AA-12

Source: Processed primary data (2021)

Presentation of Research Data on Subjects with a Sequential Concrete Thinking Style (SK)

Below are the responses from the sequential concrete thinking style subject for the mathematical reasoning test on the higher-order thinking skills (HOTS) question.

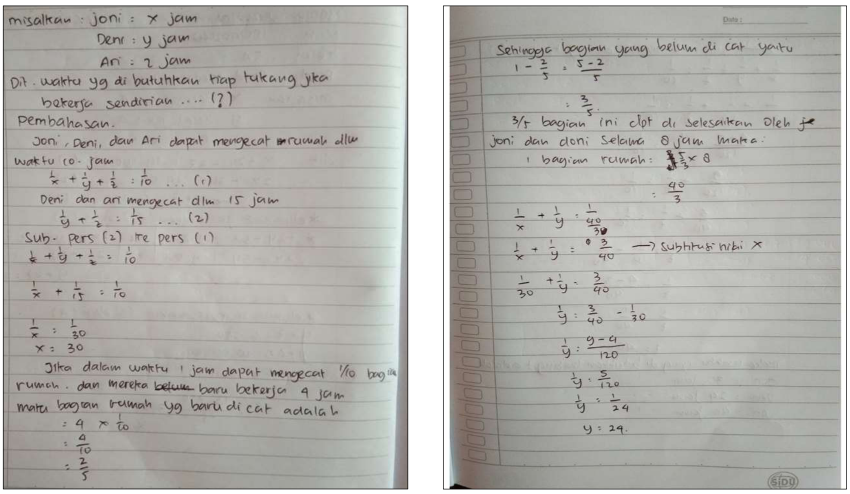

Figure 4. Mathematical Reasoning Test Responses of the Sequential Concrete Thinking Style Subject

In addressing the higher-order thinking skills problem, the subject first defines the given and asked components. Initially, the subject assigns variables x, y, and z to represent Joni, Deni, and Ari, respectively, and then formulates two linear equations with three variables based on the information provided in the problem. The first statement, which indicates that Joni, Deni, and Ari can paint the house in 10 hours, is translated into the equation $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ for the first equation. The second statement, which mentions that Deni and Ari can paint the house in 15 hours, is expressed as $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$ for the second equation. Subsequently, the subject performs substitution of the second equation into the first equation to solve for x. In this step, the subject substitutes $\frac{1}{y}+\frac{1}{z}$ with $\frac{1}{15}$ resulting in the equation $x = 30$. The subject then carefully rereads the problem and interprets the statement, "The three painters work together to paint the house for 4 hours. Afterward, Ari leaves due to an urgent matter, and Joni and Deni need an additional 8 hours to complete the painting," by stating that the three painters together can paint $\frac{1}{10}$ of the house in 1 hour. The subject further calculates that if they work for 4 hours, they can paint $4*\frac{1}{10}=\frac{4}{10}=\frac{2}{5}$ of the house. Next, the subject subtracts $\frac{2}{5}$ from the whole, leaving $\frac{3}{5}$ of the work remaining, which is completed by Joni and Deni in the following 8 hours. The subject then sets up a proportion $\frac{3}{5}$ of the work equals 8 hours of labor, leading to the equation for a full job by Deni and Ari $8*\frac{10}{6}=\frac{40}{3}\ hours$. After obtaining the time, the subject derives the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{40}{3}} \rightarrow \frac{1}{x}+\frac{1}{y}=\frac{3}{40}$. Substituting the value $x=30$ into the equation, the subject solves $\frac{1}{30}+\frac{1}{y}=\frac{3}{40}$, yielding the value of $y=24$ hours.

The subject then substitutes the values $x=30$ and $y=24$ into the first equation, obtaining $\frac{1}{30}+\frac{1}{24}+\frac{1}{z}=\frac{1}{10}$ and solves for $z=40$ hours. After determining the values of the three variables, the subject concludes by stating that the time required for each individual to complete the painting is 30 hours for Joni, 24 hours for Deni, and 40 hours for Ari, as shown in the diagram below.

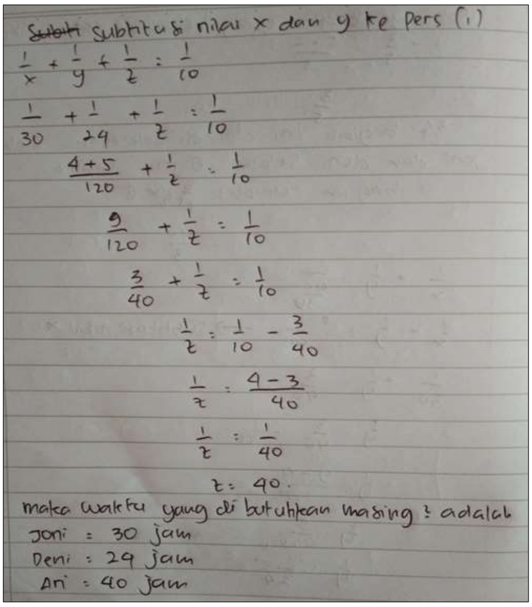

Figure 5. The subject's answer to the reasoning test reflects a concrete sequential thinking style

Based on the results of the mathematical reasoning test and the interview with the subject, who exhibits a concrete sequential thinking style, the following findings were obtained.

Tabel 3. The findings based on the results of the mathematical reasoning test and the interview with the Subject Concrete Sequential Thinking Style

Valid Data	***Kode***
Indicator for Presenting Mathematical Statements Verbally, in Writing, and through Diagrams and Figures	
The subject is able to explain the presentation in the form of a model, diagram, factors, and the properties of the relationships in the problem by representing the equation as fractions, with the reasoning that the question asks for time/hours, thus a reciprocal comparison is used. The subject then constructs two equations $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ and $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$	DV-SK-02
Indicator for Making Hypotheses or Proposing Assumptions	
The subject assigns variables to represent the times: Joni = x hours, Deni = y hours, and Ari = z hours. The subject then formulates two equations with three variables derived from the problem $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ and $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$. he subject then states that the question asks for the time it takes for Joni, Deni, and Ari to paint the house if they were working alone, which is the time each would take to complete the task individually.	DV-SK-04
Indicator for Performing Mathematical Manipulation	
The subject writes and explains the steps taken to solve the problem using the substitution method combined with reciprocal comparison. The subject constructs the mathematical model $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ and $\frac{1}{y}+\frac{1}{z}=\frac{1}{15}$ then performs substitution to obtain the value $x=30$. The subject then explains that, based on reciprocal comparison, they derived the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{40}{3}}$, After that, the subject performs further substitution, obtaining the values $y=24$ and $z=40$	DV-SK-06
Indicator for Constructing Proof and Providing Justification for the Correctness of the Solution	
The subject explains that they used the method of comparison based on the known values to derive the equation, $\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{40}{3}}$. From this, they obtained the value $y=24$. The subject then substituted the values of x and y into the equation $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{10}$ which resulted in $z=40$. Finally, the subject substituted $x=30$ into the equation, confirming that $y=24$ and $z=40$.	DV-SK-08
Indicator for Drawing Conclusions from Statements	
The subject explains that after obtaining the values for x, y, and z, they concluded that Joni requires 30 hours, Deni requires 24 hours, and Ari requires 40 hours	DV-SK-10
Indicator for Verifying the Validity of an Argument	
The subject expresses confidence, stating that they are certain of their solution, as they completed the problem carefully, step by step	DV-SK-12

Source: Processed primary data (2021)

Discussion

Based on the results of the written test and interviews, both subjects demonstrated effective responses in presenting mathematical statements, highlighting the influence of their distinct thinking styles on problem-solving approaches. The Abstract Random subject (AA) approached the task intuitively and holistically, simplifying the given information into mathematical models using broad conceptual understanding. In contrast, the Concrete Sequential subject (SK) adopted a structured and systematic approach, carefully organizing the information into coherent steps to create mathematical models. These findings align with Setiawan et al. (2020), who emphasizes that reasoning involves the coherent presentation of models, facts, properties, and relationships, reflecting how different cognitive styles shape problem-solving strategies.

These differences in making conjectures highlight the unique strengths of each thinking style in mathematical reasoning. The Abstract Random subject's preference for fractional representations reflects their ability to view problems conceptually, allowing them to explore unconventional methods and identify patterns that may not be immediately apparent (Suryaningrum et al., 2020). Conversely, the Concrete Sequential subject's structured approach to constructing a system of equations emphasizes their focus on logical consistency and precise organization, which ensures accuracy in problem-solving (Sinyukova et al., 2021). This distinction underscores the importance of understanding and leveraging individual thinking styles in educational contexts, as it can inform tailored instructional strategies that support students' natural problem-solving tendencies while addressing their specific challenges (Güngör & Baysal, 2024; Kholid et al., 2020; Tamimi et al., 2024). This understanding of the unique strengths of each thinking style reinforces the need for educators to recognize and adapt to the diverse cognitive approaches students bring to problem-solving, ultimately fostering a more inclusive and effective learning environment.

These findings collectively demonstrate how different thinking styles influence the key aspects of mathematical reasoning: constructing proof, drawing conclusions, and verifying the validity of arguments. Both subjects successfully justified their solutions, drew logical conclusions, and revisited their answers to ensure accuracy, although their approaches differed significantly. The Abstract Random subject relied on intuitive exploration and conceptual understanding, often revising their reasoning as new patterns emerged, which aligns with the flexibility described by Ayyoub & Al-Kadi (2024). Meanwhile, the Concrete Sequential subject demonstrated a structured approach, systematically building and verifying each step of their solution, consistent with Pamungkas & Masduki (2022) findings on the strengths of sequential thinkers in maintaining logical consistency. Despite these differences, both subjects exhibited the ability to construct logical proofs, derive accurate conclusions, and validate their reasoning, underscoring the adaptability of mathematical reasoning skills across diverse cognitive styles. These outcomes align with Taufik et al. (2021), who emphasizes that reasoning involves constructing proof and validating solutions, and Ayyoub & Al-Kadi (2024), who highlights the importance of connecting concepts through universally accepted rules to draw logical conclusions. These results underscore the interplay between conceptual flexibility and procedural precision, emphasizing the need for instructional strategies that foster both intuitive and systematic approaches to problem-solving.

Building on the findings regarding the reasoning indicators, a comparison of the two thinking styles further reveals significant differences in their approaches to solving Higher Order Thinking Skills (HOTS) problems. The Abstract Random subject exhibited notable strengths in flexibility, intuitive reasoning, and the ability to conceptualize problems holistically, often exploring unconventional methods to arrive at solutions. In contrast, the Concrete Sequential subject excelled in precision, logical consistency, and systematic problem-solving, favoring structured and step-by-step methodologies to ensure accuracy. These contrasting approaches underscore the need for educators to recognize and accommodate diverse cognitive styles when designing instructional strategies. By understanding these differences, educators can create more inclusive learning environments that foster both conceptual creativity and procedural rigor, enabling students to leverage their unique cognitive strengths while addressing their individual learning challenges.

Implications

The implications of this research provide valuable insights into the mathematical reasoning abilities of students, particularly in the context of solving Higher Order Thinking Skills (HOTS) problems. By examining the relationship between abstract random thinking and sequential concrete thinking styles, the findings offer a deeper understanding of how cognitive styles influence students' problem-solving approaches. These insights can serve as a foundation for designing tailored instructional strategies that cater to diverse cognitive needs in mathematics education. For instance, students with an abstract random thinking style may benefit from flexible, exploratory problem-solving tasks that encourage conceptual understanding, while those with a sequential concrete thinking style may thrive in structured, systematic environments that emphasize step-by-step reasoning.

Beyond classroom strategies, these findings have broader implications for curriculum design and teacher training programs. Educators can use this research to develop instructional materials and assessment tools that align with different cognitive styles, ensuring that all students have equal opportunities to excel in mathematics. Additionally, professional development programs can incorporate these insights to help teachers identify and address the unique needs of students with varying cognitive preferences, fostering a more inclusive and effective learning environment.

From a theoretical perspective, this research contributes to the growing body of literature on mathematical reasoning and cognitive styles, emphasizing the interplay between conceptual flexibility and procedural precision. By highlighting these dynamics, the study provides a framework for future research to explore how other thinking styles, such as concrete random or abstract sequential, interact with reasoning abilities. Ultimately, these findings can inform the development of more nuanced instructional models that support the cultivation of higher-order cognitive skills across diverse learning contexts.

Limitations and Suggestions

This study primarily focuses on mathematical reasoning abilities in subjects with abstract random and sequential concrete thinking styles, which provides valuable insights into how these cognitive styles influence problem-solving strategies. However, the research has certain limitations. First, the study involves a small sample size, which may restrict the generalizability of the findings to broader student populations. Second, the scope of the study is limited to two thinking styles, leaving other styles such as concrete random and abstract sequential unexplored. Third, the findings are context-specific and may not fully account for the influence of external factors, such as instructional methods or prior knowledge, on students' reasoning processes.

To address these limitations, future research could expand its scope by investigating the mathematical reasoning abilities of students with concrete random and abstract sequential thinking styles. Such an exploration would provide a more comprehensive understanding of how diverse cognitive styles influence reasoning and problem-solving approaches, particularly in the context of higher-order thinking tasks. Additionally, incorporating larger and more diverse samples could enhance the generalizability of findings. Future studies could also examine the role of instructional strategies, cultural factors, and prior knowledge in shaping the relationship between cognitive styles and mathematical reasoning. By addressing these aspects, future research can contribute to the development of more inclusive instructional models that accommodate a wider range of cognitive styles. Moreover, it could deepen theoretical insights into the interplay between cognitive preferences and reasoning processes, ultimately enriching the literature on mathematical reasoning and higher-order thinking.

CONCLUSION

The study concludes that students with Abstract Random and Sequential Concrete thinking styles demonstrate mathematical reasoning abilities that align with key indicators, such as presenting mathematical statements, making conjectures, performing mathematical manipulations, constructing proofs, and drawing conclusions. Both thinking styles successfully address Higher Order Thinking Skills (HOTS) problems related to systems of linear equations with two variables (SPLTV), highlighting their adaptability in reasoning processes. Abstract Random students excel in conceptual flexibility and intuitive problem-solving, often utilizing holistic interpretations, while

Sequential Concrete students demonstrate procedural accuracy and logical consistency, employing reciprocal comparisons and systematic methods. These differences underscore the role of cognitive preferences in shaping students' approaches to reasoning and problem-solving. By understanding these distinctions, educators can develop tailored instructional strategies to enhance mathematical reasoning, such as fostering exploratory tasks for Abstract Random thinkers and structured methodologies for Sequential Concrete thinkers. While this study provides valuable insights, its scope is limited to two cognitive styles and a small sample size. Future research should explore additional thinking styles, such as Concrete Random and Abstract Sequential, and examine external factors, such as instructional methods and prior knowledge, to gain a more comprehensive understanding of the relationship between cognitive styles and reasoning abilities in HOTS contexts.

AUTHOR CONTRIBUTIONS STATEMENT

All authors contributed significantly to the completion of this research. The specific contributions of each author are as follows:

1. Ma'rufi: Responsible for the conceptualization and design of the study, development of research instruments, data collection, and initial drafting of the manuscript.
2. Muhammad Ilyas: Contributed to data analysis, interpretation of results, and critical revisions to the manuscript to ensure intellectual content and clarity.
3. Nur Wahidin Ashari: Provided expertise in mathematical reasoning and thinking styles, validated the research framework, and contributed to the review of related literature.
4. Tri Bondan Kriswinarso: Assisted in data collection, organization of raw data, and preparation of visual materials, such as tables and figures.
5. Salwah: Supervised the overall research process, reviewed and refined the manuscript, and ensured alignment with journal submission requirements.

All authors have read and approved the final manuscript and agree to be accountable for all aspects of the work.

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