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Exploring the dynamics of didactic practices in ratio learning: a qualitative study of teachers' instructional strategies and student engagement in junior secondary mathematics

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Abstract

Background: The teaching of ratio remains a significant challenge in mathematics education, as students often struggle to understand multiplicative relationships and tend to rely on procedural reasoning rather than conceptual understanding. Previous studies have mainly focused on students' conceptual difficulties, while limited attention has been given to how teachers' didactic practices influence students' engagement and understanding.
Aims: This study aims to analyze the dynamics of didactical practices in ratio learning, focusing on instructional strategies, classroom interaction, and student engagement, and offers a novel contribution by examining how didactical practices shape variations in student engagement in real classroom contexts.
Method: This research employed a qualitative case study design involving one mathematics teacher and 28 seventh-grade students selected through purposive sampling. Data were collected through classroom observations, semi-structured interviews, and documentation, and analyzed using thematic analysis.
Results: The findings indicate that teaching practices were dominated by direct instruction combined with contextual examples, which supported initial understanding but limited deeper conceptual exploration. Approximately 60% of students were actively engaged, while the others tended to be passive. Student interaction was mostly procedural, and the use of a reward system increased participation but encouraged performance-oriented behavior.
Conclusion: These findings highlight the important role of didactic practices in shaping students' engagement and understanding of ratio. It is recommended that teachers adopt more student-centered approaches, such as structured discussions and contextual problem-solving, to enhance conceptual understanding and equitable participation in mathematics learning.

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INTRODUCTION

Mathematics education plays a crucial role in fostering students' logical, analytical, and critical thinking skills, which are essential for addressing complex problems in everyday life. Mathematics should not be viewed merely as a collection of procedures and formulas to be memorized; rather, it serves as a means of developing systematic reasoning and adaptive problem-solving abilities (Bakker et al., 2021b; Karp & Bay-Williams, 2020; OECD, 2024). Therefore, the quality of classroom mathematics instruction becomes a key determinant in supporting students' development of deep conceptual understanding. A growing body of research indicates that well-designed instructional practices not Only enhance learning outcomes but also strengthen students' cognitive and affective engagement in the learning process (Hillmayr et al., 2020; Stylianides et al., 2024; Thomsen et al., 2022).

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Within school mathematics, the concept of ratio is fundamental, as it underpins a wide range of subsequent topics, including proportion, percentage, scale, and linear functions. However, numerous studies have identified ratio understanding as a persistent challenge for students, particularly in grasping the multiplicative relationships between quantities (Lamon, 2020; Reinhold et al., 2020). Students often rely on additive reasoning rather than multiplicative reasoning, leading to misconceptions about ratio as a proportional relationship (Bakker et al., 2021b; Ferri et al., 2021). Recent research further highlights students' difficulties in interpreting various representations of ratios, such as tables, graphs, and contextual situations, which in turn limits their ability to transfer knowledge to new contexts (Qiu et al., 2020; Wiesner et al., 2020). These findings are further supported by recent studies showing that students continue to experience difficulties in proportional reasoning, particularly in understanding relationships between quantities and applying ratio concepts flexibly (Begolli et al., 2021; Furumoto et al., 2025; Gómez & Rico, 2025; Pişkin Tunç & Çakıroğlu, 2022).

These difficulties are not solely attributable to the inherent complexity of the concept but are also closely related to how ratio is taught in the classroom. Instruction that overemphasizes procedural knowledge without fostering conceptual understanding tends to reinforce misconceptions (Bakker et al., 2021b; Ferri et al., 2021). Consequently, it is essential to examine how classroom instructional practices, particularly teachers' didactical practices, shape students' understanding of ratio.

In this regard, teachers' instructional practices are a central factor in determining how mathematical concepts are introduced, represented, and constructed by students. In this study, didactical practice refers to the set of pedagogical actions undertaken by teachers in organizing instruction, including instructional strategies, the use of learning media, and patterns of classroom interaction (Heilporn et al., 2022; Kazemi & Hintz, 2023; Mosvold, 2024; Rahimi, 2024). These practices reflect not only teaching activities but also the pedagogical decisions that structure meaningful learning experiences. Therefore, analyzing didactical practice is essential for understanding how classroom dynamics influence both student engagement and conceptual understanding of ratio. Recent studies further highlight that instructional strategies and learning environments significantly influence students' engagement and understanding across various educational contexts (Baigi et al., 2024; Demelash et al., 2024; Heilporn et al., 2022; Rahimi, 2024).

Although numerous studies have examined the effectiveness of instructional models in improving mathematics learning outcomes, most of this research adopts experimental approaches that primarily focus on student achievement. As a result, there remains limited understanding of how instructional practices are enacted in real classroom contexts, particularly in the teaching of ratio. Moreover, research on ratio learning has largely concentrated on students' conceptual difficulties, such as misconceptions and errors in proportional reasoning (Braithwaite et al., 2021; Ferri et al., 2021; Rojo et al., 2023). However, studies that explicitly connect these difficulties with teachers' didactical practices are still scarce. This is problematic, given that instructional practices play a crucial role in shaping students' learning experiences and conceptual development. Therefore, a significant research gap exists in integrating the analysis of teachers' didactical practices with the teaching and learning of ratio, particularly in understanding how instructional strategies and classroom interaction dynamics influence student engagement. This study seeks to address this gap by examining didactical practices in the context of ratio instruction within actual classroom settings. However, there is still limited empirical evidence that explicitly explains how didactical practices influence variations in student engagement specifically in the context of ratio learning.

Based on the foregoing discussion, this study aims to analyze the dynamics of didactical practices in mathematics instruction on the topic of ratio at the junior secondary school level. This study offers a novel contribution by explicitly examining how didactical practices shape variations in

student engagement within the context of ratio learning in real classroom settings. Specifically, the study focuses on examining the instructional strategies employed by teachers, the use of learning media in explaining ratio concepts, and the nature of student engagement during the learning process. Furthermore, this study seeks to explain how teachers' didactical practices, reflected in instructional strategies and classroom interaction patterns, contribute to variations in student engagement in understanding ratio concepts. By doing so, this research is expected to bridge the gap between studies on students' conceptual difficulties and classroom instructional practices, while also providing implications for the development of more effective and meaningful mathematics teaching.

LITERATURE REVIEW

In mathematics education, the interaction between teachers and students is a fundamental element that shapes the quality of the learning process. Effective instruction is not solely determined by content delivery but also by the extent to which students actively participate in constructing knowledge. Student engagement encompasses cognitive, emotional, and behavioral dimensions, manifested through activities such as discussion, idea sharing, and active participation in problem-solving (Boaler & Dweck, 2022; Fredricks et al., 2024; Wang et al., 2022). Recent research also demonstrates that student engagement is strongly shaped by instructional design, classroom climate, and teacher positioning in learning environments (Canonigo, 2024; Cevikbas & Kaiser, 2022; Ghosh & Jana, 2025; Hettinger et al., 2023). Empirical evidence suggests that higher levels of engagement are positively associated with deeper conceptual understanding and more advanced mathematical thinking (Cai, 2023; Fredricks et al., 2024; Ölmez et al., 2025; Wang et al., 2022).

In the context of ratio learning, engagement becomes particularly critical due to the complex relational nature of the concept, which cannot be effectively acquired through procedural memorization alone. Students need to actively construct their understanding of relationships between quantities through exploration, representation, and discourse. Recent studies indicate that engagement in context-based activities and the use of multiple representations can significantly support students in developing flexible and transferable understanding of ratios (Braithwaite et al., 2021; Reinhold et al., 2020; Rojo et al., 2023; Wiesner et al., 2020). Thus, the instructional strategies employed by teachers and the ways in which classroom interactions are managed play a pivotal role in facilitating meaningful student engagement.

However, in actual classroom practice, student engagement is often unevenly distributed. While some students actively participate, others tend to remain passive and minimally involved. This disparity suggests that the dynamics of teachers' didactical practices directly influence the distribution of student engagement. Therefore, it is important to investigate how instructional strategies, learning media, and classroom interaction patterns contribute to variations in student engagement during ratio learning.

Research in mathematics education consistently emphasizes the importance of student-centered approaches in promoting deep conceptual understanding. Constructivist perspectives, for instance, position learners as active agents who construct knowledge through interaction with their learning environment (Fosnot & Perry, 2023; Vygotsky, 1978). Within this framework, teachers are not merely transmitters of knowledge but facilitators who design learning experiences that enable students to construct meaning both individually and socially. This perspective is particularly relevant to ratio learning, as the concept requires understanding relationships between quantities that cannot be directly grasped without reflection and representation. Studies have shown that instructional approaches incorporating real-world contexts, multiple representations, and classroom discourse can enhance students' conceptual understanding of ratios and promote greater flexibility in thinking (Bakker & Wagner, 2020; Braithwaite et al., 2021; Rojo et al., 2023). Recent studies also demonstrate

that contextual and realistic approaches can enhance students' conceptual understanding of ratio when combined with active student participation (Isnawan et al., 2025; Purwasi et al., 2025; Sari et al., 2024). In addition, recent studies emphasize the importance of classroom discourse in supporting mathematical reasoning and conceptual understanding, particularly through dialogic interaction and student participation (Barwell, 2025; Bishop, 2021; Chen & Chan, 2022; Ni et al., 2021; Yu & Huang, 2025). Furthermore, the integration of digital technologies has been found to support the visualization of ratio relationships and increase student engagement (Y. Dobashi et al., 2022; Hillmayr et al., 2020).

On the other hand, research on teaching practices highlights that instructional strategies significantly influence both student engagement and learning outcomes (Atoyebi & Atoyebi, 2022; Sim & Matore, 2022). From this perspective, teachers' didactical practices can be understood as a system of actions reflecting pedagogical decision-making in organizing instruction, including the selection of strategies, structuring of activities, and facilitation of classroom interactions (Chen et al., 2020; Heilporn et al., 2022; Rahimi, 2024; Suh et al., 2020). Thus, examining didactical practices is essential to gain insight into how mathematics teaching is enacted in real classroom settings.

METHOD

Research Design

This study employed a qualitative approach using a descriptive case study design to examine the dynamics of didactical practices in mathematics classroom instruction. A qualitative approach was selected because the study aims to gain an in-depth understanding of naturally occurring teaching and learning processes, including teachers' instructional strategies, the use of learning media, and students' engagement during classroom activities (Creswell & Creswell, 2018; Díez-Palomar et al., 2022; L. Nowell, 2022). Furthermore, a qualitative approach is particularly appropriate for investigating the learning of ratio, as understanding ratios involves not only procedural competence but also complex cognitive processes related to relational reasoning between quantities. Prior studies indicate that ratio understanding develops through interaction, representation, and contextualized learning experiences, thus requiring a research approach capable of capturing these dynamic processes in depth (Hillmayr et al., 2020; Reinhold et al., 2020; Siegler & Oppenzato, 2021). Accordingly, this approach enables the researcher to explore how teachers' didactical practices facilitate or constrain students' conceptual understanding of ratio within authentic classroom settings.

A case study design was adopted because the study focuses on an in-depth analysis of instructional practices within a specific classroom context. Case studies allow for a holistic exploration of educational phenomena by considering the interplay among various instructional components, including teachers, students, instructional strategies, and learning media (Díez-Palomar et al., 2021; Yin, 2018). Therefore, this design is well-suited to uncover the contextualized and nuanced dynamics of didactical practices in ratio instruction. This qualitative approach is appropriate for examining ratio learning, as it allows an in-depth exploration of classroom interactions and students' conceptual understanding, which cannot be fully captured through quantitative measures.

Participant

The participants in this study consisted of one mathematics teacher and 28 seventh-grade students. The participants were selected using purposive sampling. The selection of participants was based on their involvement in ratio learning, which is considered a critical topic in developing proportional reasoning. The selection of a single teacher and one class was conducted purposively to enable an in-depth examination of didactical practices within a specific instructional context. In

qualitative research, the primary emphasis lies not on statistical generalization but on developing a deep and contextualized understanding of the phenomenon under investigation (Creswell & Creswell, 2018; Díez-Palomar et al., 2021). Thus, the use of a single case in this study aims to produce a rich and detailed description of the dynamics of didactical practices in ratio learning. Moreover, the selected classroom context is considered representative of typical mathematics instruction at the junior secondary level, thereby allowing the findings to have a degree of transferability to similar educational contexts (Díez-Palomar et al., 2021; Lincoln & Guba, 1985). Hence, despite the limited number of participants, the study provides meaningful insights into classroom mathematics practices.

The population of this study comprised all seventh-grade students at the selected school. However, purposive sampling was employed to select a specific class based on its relevance to the research objectives. The focus on ratio as the instructional topic was based on its fundamental role in mathematics and its well-documented difficulty for students. Research consistently shows that students struggle with proportional reasoning, particularly in distinguishing between additive and multiplicative relationships within ratio contexts (Bakker et al., 2021b; Hillmayr et al., 2020). Therefore, examining instructional practices in ratio learning is essential for understanding how teaching strategies influence both student engagement and conceptual understanding. Purposive sampling enables researchers to select cases that are most informative and relevant to the research questions (Patton, 2014, 2022). By choosing a class that was actively engaged in learning ratio, this study was able to directly investigate the dynamics of didactical practices within an appropriate and meaningful instructional context.

Instrument

The primary instrument in this study was the researcher, who functioned as a human instrument responsible for collecting and analyzing the data (Creswell & Creswell, 2018). To support data collection, several complementary instruments were employed, including classroom observation sheets, semi-structured interview protocols, and instructional documentation. To ensure the validity and reliability of the data, several strategies were implemented. First, content validity of the observation and interview instruments was established through expert review involving mathematics education specialists. Second, pilot testing was conducted to refine the clarity and relevance of observation indicators and interview questions prior to the main data collection. Third, data credibility was strengthened through triangulation of sources and methods by comparing findings from observations, interviews, and documentation (M. B. Miles et al., 2018; M. P. Miles & Morrison, 2020; L. Nowell, 2022; L. S. Nowell et al., 2017). To ensure the trustworthiness of the data, this study applied credibility, transferability, dependability, and confirmability criteria (Lincoln & Guba, 1985). Member checking was also conducted to validate the accuracy of the findings, ensuring that the interpretations accurately reflected participants' perspectives. Additionally, peer debriefing and member checking were employed to enhance the credibility and trustworthiness of the findings, ensuring that interpretations were grounded in the data and accurately reflected participants' perspectives (Creswell & Creswell, 2018). This approach aligns with contemporary qualitative research practices that emphasize interpretive validity in data analysis (L. Nowell, 2022; L. S. Nowell et al., 2017).

Procedure

Data collection in this study was conducted through a series of systematic and structured stages to ensure the comprehensiveness and depth of the data obtained. The first stage involved research preparation, which included the development of research instruments, validation of these instruments, and coordination with the school and the participating teacher regarding the

implementation of the study. This stage was essential to ensure that the instruments were both methodologically sound and contextually appropriate for capturing classroom practices.

The second stage consisted of classroom observations conducted over two instructional sessions focusing on direct and inverse proportion. Each observation session lasted approximately 70 minutes. The observations were conducted in two classroom sessions to capture the dynamics of instructional strategies, classroom interaction, and student engagement during ratio learning. During these observations, the researcher adopted a non-participant role in order to preserve the natural flow of classroom instruction and minimize potential researcher interference. This approach allowed for an authentic examination of the teacher's didactical practices and students' engagement during the learning process.

The third stage involved semi-structured interviews with the mathematics teacher, conducted after the completion of the observation sessions. Each interview lasted approximately 30 minutes and aimed to elicit deeper insights into the teacher's instructional strategies, underlying pedagogical considerations, and the challenges encountered in teaching ratio concepts. Interviews were conducted after the learning sessions to gain a deeper understanding of the observed teaching practices and students' learning experiences. The use of semi-structured interviews provided flexibility for probing while maintaining alignment with the research objectives.

The final stage consisted of collecting supporting documentation, including field notes and visual records of classroom activities. These documents served to complement and triangulate the data obtained from observations and interviews. Overall, the data collection process was carried out within a carefully planned timeframe to ensure data completeness, consistency, and analytical richness.

Analysis plan

The data were analyzed using thematic analysis, following the procedures outlined by (Braun & Clarke, 2006), which include data familiarization, initial coding, theme generation, theme review, theme definition, and interpretation (Braun & Clarke, 2006; L. S. Nowell et al., 2017). The analysis was conducted on data obtained from classroom observations, interviews, and documentation, focusing on patterns related to instructional strategies, classroom interaction, and student engagement in ratio learning. In this study, the analysis extended beyond mere theme identification by linking emerging themes to relevant theoretical frameworks, particularly those related to didactical practices and student engagement in mathematics learning. For instance, themes related to the dominance of direct instruction were interpreted through the lens of teaching practice theory (Schoenfeld, 2011), while themes concerning student engagement were analyzed using engagement theory (Fredrick et al., 2022; Wang et al., 2022).

For example, observational data indicating passive student behavior were interpreted as evidence of limitations in didactical practices in facilitating active engagement, which was further analyzed from a constructivist perspective (Fosnot & Perry, 2023; Vygotsky, 1978). Thus, the analysis in this study is both interpretive and theoretically grounded, rather than purely descriptive. This analytical approach is consistent with contemporary qualitative research, which emphasizes the integration of empirical data and theoretical frameworks to generate meaningful scholarly contributions (Díez-Palomar et al., 2021; L. S. Nowell et al., 2017).

RESULTS AND DISCUSSION

Results

The findings of this study were derived from classroom observations of mathematics instruction on the topic of ratio in a seventh-grade classroom. This study specifically aimed to analyze

the dynamics of teachers' didactical practices, including instructional strategies, the use of learning media, and student engagement in understanding ratio concepts. Therefore, the presentation of findings not only describes classroom activities but also interprets how these didactical practices contribute to students' understanding and engagement in ratio learning. Based on the observations, the instructional dynamics can be categorized into four main aspects: (1) the teacher's presentation of ratio concepts, (2) student interaction during the learning process, (3) student participation in problem-solving activities, and (4) variations in student engagement. These four aspects are interconnected and collectively shape the didactical practices that influence how students construct their understanding of ratio.

Teacher's Presentation of Ratio Concepts

The lesson began with an opening activity that included greeting, prayer, and an apperception phase aimed at connecting prior knowledge to the new topic. At this stage, the teacher attempted to activate students' prior knowledge as a foundation for learning new concepts. Subsequently, the teacher introduced the concept of direct proportion through explicit instruction, supported by contextual examples drawn from students' everyday experiences, such as the relationship between quantity and total price or distance and travel time.

Figure 1. Teacher explaining direct proportion

The figure illustrates the teacher delivering explanations at the front of the classroom using the whiteboard as the primary instructional medium. The teacher presented example problems and explained solution procedures step-by-step, while students listened attentively and took notes. From a didactical perspective, this activity reflects a structured and teacher-centered instructional approach emphasizing explicit concept delivery. This approach supported students in developing an initial understanding of relationships between quantities in ratio contexts. However, the dominance of direct instruction indicates that learning remained largely teacher-centered, limiting opportunities for students to independently explore the concept. As a result, students' understanding tended to be procedural, focusing on how to solve problems rather than fully grasping the relational meaning of ratios. In the subsequent lesson, the teacher introduced inverse proportion using a similar approach, combining direct explanation with contextual examples.

Figure 2. Teacher explaining inverse proportion

The figure shows how the teacher used contextual illustrations to explain the inverse relationship between variables. Although contextualization helped students develop intuitive understanding, the instructional approach still did not fully encourage students to actively construct conceptual knowledge. This finding highlights the critical role of didactical practices in determining the depth of students' understanding of ratio.

Student Interaction in the Learning Process

In addition to teacher explanations, student interaction was an important component of the classroom dynamics. Interaction primarily occurred when students were asked to work on worksheet problems and discuss their solutions with peers.

Figure 3. Students discussing problem solutions

The figure depicts students engaging in peer discussions to understand solution procedures. These discussions allowed students to exchange ideas, clarify their understanding, and compare different solution strategies. Analytically, this interaction reflects elements of social knowledge construction, where students actively engage in meaning-making through communication with peers. However, the quality of these discussions remained largely procedural, focusing on obtaining correct answers rather than exploring the underlying conceptual structure of ratios. This suggests that although student interaction was present, the teacher's didactical practices had not yet fully guided discussions toward deeper conceptual understanding.

Student Participation in Problem Solving

Student participation was evident through activities such as solving problems, consulting the teacher for answer verification, and presenting solutions in front of the class.

Figure 4. Student consulting the teacher for answer verification

The figure shows direct interaction between a student and the teacher during answer checking. This activity provided immediate feedback, which is essential for correcting errors and reinforcing understanding. Additionally, the teacher implemented a reward system by giving stickers to students who solved problems correctly and quickly. From an analytical perspective, this reward system effectively increased students' motivation and participation. Students appeared more eager to complete tasks and engage in classroom activities. However, it also potentially fostered a performance-oriented mindset, where students prioritized speed and correctness over the depth of mathematical reasoning. This finding indicates that reinforcement strategies within didactical practices can significantly influence the nature of student participation.

Variations in Student Engagement

Another important finding concerns the variation in student engagement during the learning process. Engagement was not evenly distributed; while some students actively participated in discussions and problem-solving, others remained passive.

Figure 5. Student presenting solution on the whiteboard

The figure illustrates an active student presenting their solution on the board. This activity reflects a high level of engagement, as the student not only understood the material but was also able to communicate mathematical ideas. Nevertheless, not all students demonstrated similar levels of engagement. This variation was influenced by several factors, including the teacher-centered instructional approach, differences in students' prior abilities, and unequal opportunities for participation. Overall, these findings indicate that teachers' didactical practices play a crucial role in shaping the distribution of student engagement. Instruction that does not fully provide opportunities for exploration and active participation tends to result in uneven engagement among students.

Table 1. Summary of Findings

Aspect	Key Findings	Implications
Concept delivery	Dominance of direct instruction	Supports initial understanding but limits conceptual depth
Student interaction	Discussion focused on procedures	Aids problem-solving but not conceptual understanding
Participation	Influenced by reward system	Increases motivation but promotes performance orientation
Engagement	Uneven distribution	Influenced by teaching strategies and student differences

Table 1 synthesizes the main findings across the four aspects of didactical practice observed in ratio instruction. Overall, the findings suggest that the instructional practices were predominantly characterized by direct instruction, which facilitated initial understanding but did not fully support the development of deep conceptual knowledge. Student interactions, although present, were largely procedural in nature and did not optimally promote relational understanding of ratios. The use of a reward system enhanced student motivation and participation but tended to shift students' focus toward performance outcomes rather than meaningful mathematical thinking. Meanwhile, variations in student engagement indicate that not all students had equal opportunities to participate actively, influenced by instructional strategies, differences in prior knowledge, and classroom interaction dynamics. Taken together, these findings underscore the significant role of teachers' didactical practices in shaping both student engagement and the quality of conceptual understanding in ratio learning. This highlights the need for more student-centered instructional approaches to enhance the effectiveness and meaningfulness of mathematics learning.

Discussion

The findings of this study indicate that mathematics instruction on ratio remains predominantly characterized by direct instructional strategies, complemented by the use of contextual examples. This suggests that the teacher attempts to facilitate students' understanding by simplifying concepts through familiar contexts; however, the approach largely positions students as passive recipients of information. From the perspective of teaching practice theory, this reflects a transmissive form of didactical practice, in which pedagogical decisions prioritize the efficient delivery of content rather than the active construction of student understanding (Baigi et al., 2024; Borko et al., 2008; Heilporn et al., 2022; Rahimi, 2024; Schoenfeld, 2011). When situated within recent research, these findings are consistent with (Hillmayr et al., 2020), who argue that instruction overly focused on procedural knowledge tends to produce shallow and inflexible understanding of ratio. Similarly, (Bakker et al., 2021b) emphasize that meaningful understanding of ratio requires students' active engagement in exploring relationships between quantities, rather than merely following algorithmic procedures. More recent studies further support this argument by demonstrating that conceptual understanding in mathematics is significantly enhanced when students are provided with opportunities for sense-making, reasoning, and inquiry-based learning (Cevikbas & Kaiser, 2022; Hettinger et al., 2023; Siegler & Lortie-Forgues, 2017; Wang et al., 2022; Weinberg, 2019).

Importantly, these findings are further supported by recent studies indicating that ratio remains one of the most challenging mathematical concepts for students, particularly due to its reliance on relational reasoning and the ability to interpret multiple representations (Begolli et al., 2021; Isnawan et al., 2025; Pişkin Tunç & Çakıroğlu, 2022; Sari et al., 2024). Therefore, the dominance of direct instruction observed in this study can be interpreted as a limiting factor in fostering deeper conceptual understanding. On the other hand, the use of contextual examples indicates an effort to connect mathematical concepts with students' real-life experiences. This finding aligns with the principles of Realistic Mathematics Education (RME), which highlight the importance

of context in supporting mathematical meaning-making (Gravemeijer & Doorman, 1999). However, contemporary studies suggest that the mere inclusion of context is insufficient if it is not accompanied by opportunities for conceptual exploration and student-driven inquiry (Bakker & Wagner, 2020; Gómez & Rico, 2025; Siegler & Oppenzato, 2021; Wiesner et al., 2020). In line with this, recent studies have demonstrated that the use of contextual approaches and Realistic Mathematics Education (RME) can significantly enhance both conceptual understanding and student engagement when implemented in a more student-centered and exploratory manner (Baigi et al., 2024; Demelash et al., 2024; Purwasi et al., 2025; Reza et al., 2026). In the present study, however, contextual examples primarily functioned as illustrative tools rather than as vehicles for exploration, indicating that their pedagogical potential was not fully realized.

Furthermore, the findings related to student interaction reveal that peer discussions contributed to problem-solving processes but remained largely procedural in nature. This suggests that the interactions observed did not fully support the development of conceptual understanding. From a constructivist perspective, effective learning environments should provide opportunities for students to construct knowledge through meaningful social interaction (Fosnot & Perry, 2023; Vygotsky, 1978). Recent empirical studies reinforce this view, demonstrating that mathematically rich discussions, particularly those involving reasoning, justification, and argumentation, can significantly enhance students' understanding of ratio and proportional reasoning (Bishop, 2021; Chen & Chan, 2022; Hidi & Renninger, 2026; Ni et al., 2021; Siegler & Oppenzato, 2021; Wiesner et al., 2020). Additionally, recent evidence suggests that the integration of interactive digital media can further support students' understanding of ratio concepts and enhance engagement, particularly through dynamic visual representations and flexible access to learning materials (Aulia & Pramuditya, 2025; K. Dobashi et al., 2022). Thus, the findings imply that teachers' didactical practices need to more deliberately scaffold and direct student interactions toward conceptual discourse while also leveraging appropriate learning media.

In terms of student participation, the implementation of a reward system was found to increase motivation and short-term engagement. This aligns with motivational theories suggesting that external reinforcement can enhance participation in learning activities (Ryan & Deci, 2020). However, more recent research highlights that an overreliance on extrinsic motivation may undermine deeper cognitive engagement if it is not balanced with support for intrinsic motivation (Canonigo, 2024; Ghosh & Jana, 2025; Hidi & Renninger, 2026; Jupri & Drijvers, 2016; Wang et al., 2022). In this study, the reward system appeared to encourage students to prioritize speed and accuracy, potentially at the expense of reflective and meaningful mathematical thinking. This finding suggests that while reinforcement strategies can be effective in promoting participation, they must be carefully designed to support, rather than hinder, deeper learning processes. Another important finding concerns the variation in student engagement, which was found to be uneven across the classroom. This indicates that the observed didactical practices were not fully responsive to students' diverse abilities and learning needs. Previous research has shown that student engagement is shaped by a complex interaction between instructional strategies, individual learner characteristics, and classroom environment (Fredricks et al., 2024; Hettinger et al., 2023; Wang et al., 2022). Therefore, more adaptive and student-centered instructional approaches are needed to ensure equitable participation and engagement among all students.

Overall, this study contributes to bridging the gap between research on students' conceptual difficulties in ratio learning and the analysis of classroom instructional practices. While prior studies have predominantly focused on identifying students' misconceptions and challenges (Hillmayr et al., 2020; Siegler & Lortie-Forgues, 2017), the present study highlights that these difficulties are also closely linked to teachers' didactical practices. In this sense, improving ratio learning requires not only well-designed instructional materials but also a critical transformation of teaching practices,

including the use of interactive strategies, meaningful discourse, and inclusive engagement. Nevertheless, this study has certain limitations, particularly its focus on a single classroom and one teacher. Although the case study approach allows for an in-depth and contextually grounded analysis, the findings are not intended for statistical generalization. Instead, they aim to provide a rich and nuanced understanding of the dynamics of didactical practices in mathematics instruction. Future research could expand this investigation across multiple classrooms and contexts to further validate and extend these findings.

Implications

The findings of this study have important implications for mathematics teaching practices, particularly in the instruction of ratio at the junior secondary school level. From a practical perspective, the results indicate that didactical practices still dominated by direct instruction need to be shifted toward more student-centered approaches. Teachers are encouraged to provide greater opportunities for students to actively engage in learning through discussion, exploration of contextual problems, and the use of multiple representations. Such approaches can support students not only in mastering procedures but also in developing deeper conceptual understanding. From a pedagogical perspective, these findings highlight the importance of managing classroom interactions in ways that promote equitable student engagement. The observed variation in student engagement suggests that the instructional strategies employed have not fully accommodated differences in students' abilities and learning needs. Therefore, teachers need to develop more inclusive strategies, such as problem-based learning, structured discussions, and the use of interactive media, to enhance the quality of interaction and participation in ratio learning. In addition, the use of reward systems should be more carefully designed to balance extrinsic motivation with the development of students' intrinsic motivation in learning mathematics (Ryan & Deci, 2020; Wang et al., 2022).

From a theoretical perspective, this study contributes to strengthening the understanding that students' difficulties in learning ratio are not solely due to the inherent complexity of the concept, but are also influenced by teachers' didactical practices in organizing instruction. This finding extends previous research that has primarily focused on students' conceptual difficulties (Hillmayr et al., 2020; Siegler & Lortie-Forgues, 2017), by demonstrating that instructional strategies, classroom interactions, and patterns of student engagement are critical factors shaping mathematical understanding. Thus, this study contributes to bridging cognitive and pedagogical perspectives in ratio learning.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting the findings. First, the study was conducted in a single classroom with one teacher; therefore, the findings are not intended to be generalized broadly. However, the case study design enabled an in-depth analysis of didactical practices within a specific instructional context, providing rich insights into the dynamics of ratio learning in the classroom. Second, the limited duration of observation, which was restricted to two instructional sessions, may have affected the completeness of the data. The observed practices may not fully represent the teacher's overall instructional patterns over a longer period. Additionally, the limited number of observed interactions may have influenced the depth of analysis regarding variations in student engagement. Third, this study focused on observation and interviews without incorporating direct quantitative measures of student learning outcomes. As a result, the relationship between didactical practices and improvements in students' conceptual understanding of ratio could not be empirically measured in terms of learning gains. This limitation indicates that the findings are primarily interpretative and context-bound.

Based on the findings and limitations of this study, several recommendations can be proposed for future research. First, future studies are encouraged to involve a larger number of participants

and more diverse school contexts in order to provide a more comprehensive understanding of didactical practices in ratio instruction. Comparative studies across different classes or schools may also be conducted to identify a broader range of instructional variations. Second, future research may integrate qualitative and quantitative approaches (mixed methods) to examine the relationship between didactical practices and student learning outcomes more thoroughly. This would enable not only a deeper understanding of the learning process but also the measurement of its impact on students' conceptual understanding of ratio. Third, further studies may explore the implementation of more innovative and student-centered instructional strategies, such as problem-based learning, contextual approaches, and the integration of digital technology in ratio instruction. Recent research indicates that the use of visual representations and interactive media can support students in developing more flexible and deeper understanding of ratio relationships (Siegler & Lortie-Forgues, 2017; Weinberg, 2019). Finally, future research may examine more specifically the role of social interaction and classroom discourse in supporting the development of ratio understanding, thereby contributing more substantially to the advancement of mathematics education theory, particularly in relation to classroom-based instructional practices.

CONCLUSION

This study highlights that teachers' didactical practices in ratio instruction play a critical role in shaping both student engagement and conceptual understanding. The predominance of direct instruction was found to support students in developing an initial understanding of ratio; however, it has not fully facilitated the development of deeper conceptual understanding. Although student interaction and the use of contextual examples offer significant potential to enhance understanding, these elements have not yet been optimally leveraged within classroom practice. Furthermore, the implementation of a reward system was shown to increase student participation, yet it also carries the risk of promoting a performance-oriented approach to learning rather than fostering meaningful mathematical reasoning. The observed variation in student engagement suggests the need for more inclusive and student-centered instructional strategies. Therefore, it is essential to develop instructional practices that encourage conceptual exploration, meaningful classroom discourse, and active student participation. Such approaches are crucial for improving the quality of mathematics learning, particularly in supporting students' understanding of ratio.

AUTHOR CONTRIBUTIONS STATEMENT

Azis was responsible for the research design, data collection, data analysis, and manuscript writing. Azis and Jarnawi Afgani Dahlan contributed to research supervision, the development of the conceptual framework, and the substantive revision of the manuscript. Jarnawi Afgani Dahlan also contributed to the validation of the research methodology, interpretation of the research findings, and the final refinement of the manuscript.

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