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Middle school students' mathematical representations in mathematics in context tasks on computing chances based on realistic mathematics education

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Abstract

Background: Mathematical representation plays a crucial role in students' understanding of probability concepts, particularly in computing chances, which requires coordination among visual, symbolic, and verbal forms. Previous studies indicate that students often rely on procedural calculations without adequately constructing or connecting representations, leading to shallow conceptual understanding. Realistic Mathematics Education (RME) and Mathematics in Context (MiC) tasks offer potential to address this issue by emphasizing contextual modeling and progressive formalization.
Aims: This study aims to describe the forms of mathematical representations used by junior high school students and to analyze the interrelationships among visual, symbolic, and verbal representations when solving MiC tasks on computing chances based on RME principles.
Method: A qualitative descriptive approach was employed involving 19 eighth-grade students who had participated in RME-based probability instruction. Data were collected through MiC computing chances tests adapted from Holt et al. and analyzed using the Miles and Huberman model, encompassing data reduction, data display, and conclusion drawing. This percentage is obtained from the rubric score on students' written responses per item which is then averaged.
Results: The results showed that visual representation achieved the highest level (60%), followed by symbolic representation (54%), while verbal representation was the weakest (48%). Visual models functioned effectively as model-of contexts, and symbolic representations developed through progressive formalization; however, integration among representations was not optimal due to limited verbal explanation.
Conclusion: RME-based MiC tasks effectively support the development of students' visual and symbolic representations in probability learning. Nevertheless, strengthening students' verbal representation and reflective communication is essential to enhance the coherence and meaningful integration of multiple mathematical representations.

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INTRODUCTION

Probability is widely recognized as one of the mathematical topics that students find difficult to understand. Research findings (He, 2025), many students struggle with probability problems that require them to determine the sample space and list possible outcomes, especially in combined experiments where the number of outcomes must be calculated systematically. Studies (Awuah & Ogbonnaya, 2020), also show that students often fail to interpret representations such as tree diagrams or contingency tables correctly, even when they are able to draw them, indicating limited conceptual understanding of probabilistic relationships. Furthermore, research (Iversen & Nilsson, 2019), has found that students struggle to construct visual models independently to represent the sample space of probabilistic situations, suggesting that representation plays a crucial role in

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supporting probabilistic reasoning. These findings indicate that students' difficulties in probability are not merely computational but are closely related to their limited ability to construct and coordinate multiple representations that make the structure of probabilistic events explicit.

Within this context, mathematical representation ability plays a central role in helping students organise information, connect intuitive reasoning with formal mathematical structures, and construct meaningful interpretations of probabilistic situations. Representations allow students to translate contextual information into mathematical forms such as verbal descriptions, symbolic notation, visual diagrams, or tabular structures (Goldin, 2020; Hitt, 2002; Pape & Tchoshanov, 2001). Through these representations, students can identify patterns among outcomes, clarify relationships between events and the sample space, and reason about probability values in a structured way. Flexibility in using and coordinating multiple representations therefore reflects the depth of students' mathematical understanding and contributes significantly to successful problem solving (Lesh & Harel, 2003). In probability learning, particularly in computing chances contexts that emphasize contextual reasoning and modeling, representations such as tree diagrams, sample space listings, and frequency tables function not only as calculation tools but also as models that make the structure of probabilistic events explicit (Batanero & Carmen, 2012; Jones et al., 1995).

One contributing factor to students' weak representational ability is mathematics instruction that overemphasises procedures and symbolic manipulation. Such instruction tends to direct students toward memorising algorithmic steps rather than constructing meaning from mathematical structures. Hiebert et al., (1997) asserted that procedure-oriented instruction often produces shallow understanding and misconceptions, while limiting students' opportunities to connect multiple representations. Similarly, Stein et al. (1996) emphasised that algorithm-focused learning restricts students' engagement with visual, contextual, and graphical representations that support conceptual understanding. Therefore, an instructional approach is needed that enables students to construct their own mathematical representations through meaningful and contextual learning experiences.

Realistic Mathematics Education (RME) offers such an approach by positioning real-life contexts as the starting point of mathematical learning. Through RME, students are encouraged to model real situations mathematically and gradually develop these informal models into formal mathematical representations (Freudenthal, 1991). This process allows students to meaningfully connect contexts, representations, and formal concepts.

Theoretically, mathematical representation is understood as both an internal and external system used by students to express, process, and make sense of mathematical ideas. External representations include observable symbols, images, graphs, tables, and diagrams, whereas internal representations relate to students' mental structures and cognitive processes in constructing mathematical meaning (Goldin, 2018). Representation does not merely function as a communication tool, but also constitutes an integral component of mathematical thinking that shapes students' reasoning and decision-making when solving problems. In line with this perspective, the National Council of Teachers of Mathematics positions representation as one of the essential process standards in mathematics education, emphasizing that mathematical ideas can be expressed through various forms and that the way ideas are represented fundamentally influences how they are understood and used (Principles and Standards for School Mathematics, 2000).

Empirical studies consistently indicate that students struggle to represent probabilistic situations, particularly in connecting contextual problems to appropriate mathematical models. Batanero et al. (2016) found that many students tend to apply probability formulas directly without first constructing the sample space, resulting in conceptual errors. Similar findings were reported by Garfield & Ben-zvi (2008) who noted that weaknesses in visual and symbolic representations often lead to procedural and unsustainable understanding. Other studies have shown that students struggle to construct visual representations such as tables and probability diagrams, even when they are capable of performing symbolic computations (Arifah et al., 2020; Rohmatin et al., 2025). These difficulties persist even at the higher education level, where prospective mathematics teachers still exhibit weaknesses in visual and verbal representations in probability learning (Supriadi & Ningsih, 2022). In addition, Post & Prediger (2024) reported that students encounter challenges in translating contextual situations of uncertainty into appropriate mathematical representations. Collectively, these findings suggest that representation is a central issue in students' understanding of probability concepts.

The RME approach emphasises mathematization, which includes transforming real-world situations into mathematical models (horizontal mathematization) and refining these models into more formal representations (vertical mathematization) (Gravemeijer, 1994). Through this process, representations evolve from model-of contextual situations into model-for mathematical reasoning, supporting both conceptual understanding and problem-solving competence (Treffers, 1987).

Consistent with these principles, Mathematics in Context (MiC) tasks provide a powerful instructional medium for developing students' mathematical representation abilities. MiC tasks are designed to situate mathematical learning in realistic contexts that are familiar to students, encouraging them to construct informal representations such as diagrams, tables, or sketches before progressing to formal symbolic notation. This aligns with the view of mathematics as a human activity, where meaning is constructed through modelling real situations (Freudenthal, 1991). In probability learning, MiC tasks support students in understanding uncertainty and event relationships through visual and contextual representations, such as tree diagrams and probability tables, before formalization. Previous studies have shown that contextual modelling within RME enhances students' representational quality and reduces purely procedural reasoning in probability learning (Garfield & Ben-zvi, 2008; Heuvel-panhuizen, 2003).

Previous studies have examined students' mathematical representation abilities in probability; however, most of these studies primarily focus on measuring the level or product of representations rather than analyzing the processes through which representations are constructed and coordinated. For instance, Arifah et al., (2020), reported that students tend to rely on a single representation when solving counting rule problems in probability, with high-ability students able to use both visual and symbolic forms while lower-ability students mainly rely on symbolic calculations. Similarly, Rohmatin et al. (2025), analyzed students' representational abilities in probability word problems based on learning styles and found that students predominantly produced visual representations and symbolic equations, but the study mainly categorized the types of representations that appeared. In addition, Post & Prediger (2024) found that students' representation ability in solving PISA-type problems on uncertainty and data was generally low, particularly in visual and symbolic representations, while Supriadi and Ningsih (2022), reported that university students' representation ability in probability distribution topics was also relatively low, especially in visual and verbal representations. These studies provide important evidence about the level and types of representations students produce, yet they largely rely on test results or descriptive categorization and therefore remain product-oriented rather than process-oriented.

However, research that specifically examines how junior high school students construct, translate, and connect mathematical representations in probability remains limited, particularly in tracing the transition from context-based representations (model-of) to more general mathematical representations (model-for) within Realistic Mathematics Education (RME). Few studies analyze how students coordinate visual, symbolic, and verbal representations when working on contextual tasks such as computing chances problems in Mathematics in Context. This gap highlights the need for in-depth qualitative analysis of students' representational processes in an RME-based probability learning context.

Accordingly, this study aims to analyse how students construct mathematical representations when solving Mathematics in Context (Mic) tasks in the computing chances content based on Realistic Mathematics Education (RME). Specifically, this study seeks to address the following research questions: (1) How do students construct verbal, visual, and symbolic mathematical representations when solving MiC tasks in the computing chances content based on RME?; (2) How are the interrelationships among these mathematical representations formed in students' understanding of probability concepts?

METHOD

Research Design

This study aims to identify the forms of mathematical representations used by students in solving Mathematics in Context (MiC) tasks in the computing chances content based on Realistic Mathematics Education (RME), and to analyze the interrelationships and development of students' representations during the process of solving probability problems. To address these objectives, this study employed a qualitative descriptive approach that emphasises an in-depth interpretation of students' representational processes. Quantitative information, such as scores and percentages, was used only as supporting descriptive data to illustrate general response patterns, while the main analysis focused on exploring the characteristics, connections, and development of students' mathematical representations based on their written work and explanations, without manipulating variables or testing group differences (Creswell, 2016).

Participants

The participants in this study were 19 eighth-grade students from a junior high school in Indonesia who had previously received instruction on probability through a learning approach based on Realistic Mathematics Education (RME). The students were typically 13–14 years old and were enrolled in a class where probability topics had been taught using contextual problems and mathematical modeling activities consistent with RME principles. Participants were selected purposively to ensure that they had experienced instruction aligned with the characteristics of RME, particularly the use of contextual situations and the development of models from model-of to model-for representations (Gravemeijer & Doorman, 1999). The selection criteria included students who had completed the probability learning unit and were willing to participate in the study. The study was conducted with permission from the school administration and the mathematics teacher, and informed consent was obtained from participants prior to data collection.

Instruments

This study was conducted in three main stages. The first stage involved preparation, including the development of probability learning materials based on Realistic Mathematics Education (RME) and the adaptation of the Mathematics in Context (MiC) Computing Chances test developed by Holt et al., (2006) into Indonesian. The adaptation process followed several systematic steps to maintain the integrity of the original constructs. Initially, the test items were translated from English into Indonesian. The translated version was then reviewed by a panel of mathematics education experts (mathematics education lecturers) to evaluate linguistic clarity, contextual relevance, and alignment with the intended probability concepts. Revisions were made based on the experts' feedback to improve the clarity of wording and the suitability of the items for junior high school students. The revised instrument was subsequently piloted with a small group of students who had characteristics similar to the study participants to identify potential ambiguities or difficulties in understanding the items. Feedback from the pilot test was used to refine several contextual expressions while preserving the conceptual structure of the original tasks.

The second stage involved the implementation of instruction, in which probability learning emphasized the use of realistic contexts and exploratory activities consistent with the principles of Realistic Mathematics Education (RME) to support students' conceptual understanding of probability. The final stage consisted of data collection and analysis, which were conducted by administering the adapted MiC Computing Chances test to the students. The specifications of the test instruments used in this study are presented in the following table.

Table 1. Test Specification

No Items	Probability Indicators	Mathematical Representation Assessed	Characteristics of RME
1	Explaining compound event probabilities using an area model	• Verbal: explaining the relation of diagram and probability • Visual: area model • Symbolic: Interpreting fractional values in the area model	• Model of • Model for • Progressive Formalization
2	Determining the probability of compound events and the combined event "only one occurs"	• Visual: completing probability tree • Symbolic: fractional probability calculations • Verbal: explaining strategies	• Model of • Model for • Progressive Formalization • Reflection
3	Determining the probability of compound events	• Visual: area model and probability tree diagram • Symbolic: probability on every branch • Verbal: explaining area model	• Model of • Model for • Intertwinement inter representation • Reflection
4	Calculating probabilities in repeated trials with two possible outcomes	• Visual: probability tree • Symbolic: multiplication of probabilities, fractions, and percentages • Verbal: explaining calculation	• Model of • Model for • Progressive formalization • Reflection on results
5	Determining the probability in repeated trials	• Visual: probability tree • Symbolic: probability exponentiation • Verbal: explaining how a probability is obtained	• Model of • Model for • Progressive formalization Reflection on results
6	Probability in repeated trials with more than two possible outcomes	• Visual: probability tree • Symbolic: multiplication, addition, and complementary probabilities • Verbal: Explanation and reflection on the use of the area model	• Model of • Model for • Progressive formalization • Reflection on the results

Data Analysis

Data analysis used Miles dan Huberman model (Fadilah & Hakim, 2022), consisting of data reduction, data display, and conclusion drawing and verification. In the data reduction stage, students' responses were selected and coded based on indicators of probability content, types of mathematical representations (verbal, visual, and symbolic), and RME characteristics. The following are the indicators observed in the RME elements.

Table 2. Analytical Coding Framework

RME Construct	Observable Indicators in Students' Work
Model-of	Students construct or use visual models (e.g., area models or probability trees) that directly represent the contextual situation described in the problem.
Model-for	The model begins to function as a general tool for reasoning about probabilities beyond the specific context, such as using the probability tree structure to compute other compound events.
Progressive formalization	Students move from contextual reasoning toward formal probability calculations using symbolic expressions such as fractions, multiplication rules, or exponentiation.

RME Construct	Observable Indicators in Students' Work
Intertwinement of representations	Students coordinate multiple representations (visual, symbolic, and verbal) to justify probability reasoning.
Reflection	Students evaluate, justify, or reconsider their solutions, often explaining why a particular model or strategy is appropriate.

The data display stage was carried out using tables and graphs to present the scores and percentage levels of representation achievement for each test item. Students' written responses were analyzed using the scoring guidelines presented below to determine the extent to which each indicator was demonstrated in their work.

Table 3. Scoring

Score	General Criteria
0	no relevant answer is given, or all solution step reflects a misunderstanding of probability concepts.
1	The response is very limited; probability ideas are present but inaccurate or yield an incorrect final result. Representations are unclear and show no meaningful connections.
2	The answer is mostly correct in terms of result, but the solution steps are incomplete. The student does not explain the origin of the probability values, and the connection to the visual model is not evident.
3	The response is correct and relatively complete. Probability operations are accurate and the final result is appropriate, but verbal explanation or model reflection remains limited or inconsistent.
4	The response is complete, accurate, and coherent, demonstrating strong coordination between context, probability tree (model-of/model-for), and symbolic computation, accompanied by clear explanation and reflection consistent with RME principles.

The achievement score for each item was converted into a percentage using the following formula:

$$\text{Percentage of Achievement} = \frac{\text{Total score obtained}}{\text{Maximum possible score}} \text{ x } 100\%$$

Subsequently, conclusion drawing and verification were conducted by examining the consistency of emerging patterns to obtain a comprehensive picture of students' mathematical representation abilities within RME-based probability instruction.

RESULTS AND DISCUSSION

Results

The Computing Chances Test (Holt et al., 2006) was designed to assess students' understanding of probability through the integration of visual, symbolic, and verbal representations within an RME-based learning context. Each task utilises visual models, such as area models and probability tree diagrams, as models of contextual situations that have the potential to develop into mathematical reasoning models through progressive formalization. The analysis also examines the representational trajectories within students' problem-solving processes, focusing on how students transition between representations and how these connections support or hinder their reasoning. Particular attention is paid to typical pathways, successful representational coordination, and failures within representational relationships. These patterns are identified through iterative analysis of students' written responses and explanations. Therefore, the following sections present both overall achievement levels for each representation type and qualitative patterns that characterize students' representational trajectories.

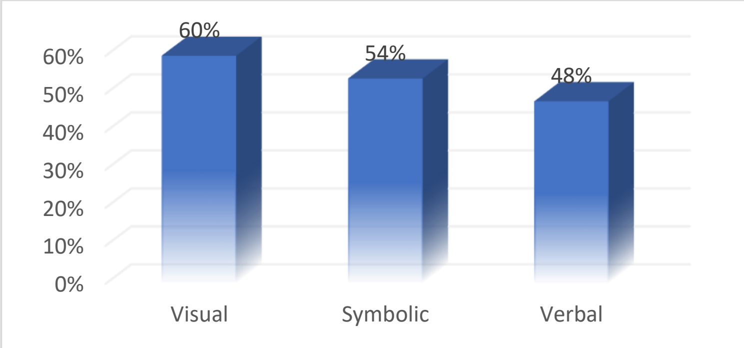


Figure1. Percentage of Mathematical Representation Achievement

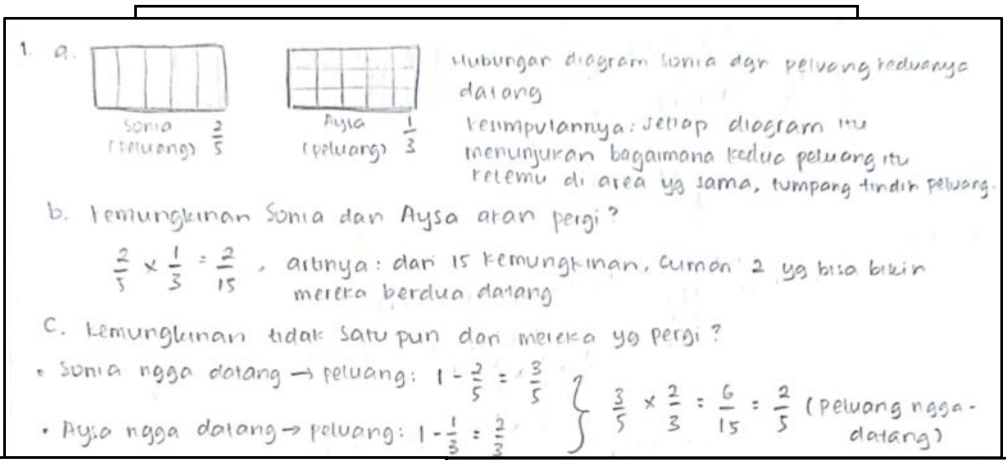
Based on Figure 1, the graph shows that visual representation (60%) attained the highest achievement level, as all items consistently positioned visual models, such as area models and probability tree diagrams, as the starting point (model-of) for understanding probabilistic situations. Symbolic representation reached 54%, requiring students to compute probabilities through multiplication, addition, exponentiation, and complementary events as part of the progressive formalization process. Meanwhile, verbal representation (48%) exhibited the lowest performance, indicating students' difficulties in explaining relationships among models, articulating strategies, and reflecting on outcomes, despite these aspects being explicitly required in almost all items through RME characteristics such as reflection and intertwinement. This pattern indicates that students were relatively strong in using visual models, reasonably capable in symbolic manipulation, yet still weak in articulating their understanding verbally in accordance with the demands of RME-based instruction.

Table 4 presents students' achievement for each item based on the scoring rubric used to evaluate their written responses. Each item was scored on a four-point scale (0–4) according to the extent to which the expected representation and reasoning were demonstrated in the student's work. The mean score represents the average score obtained by 19 students for each item. For ease of interpretation, the mean score was converted to a percentage of the maximum possible score. Therefore, the percentage indicates the relative level of achievement for each item, allowing for comparison of student performance across different probability tasks.

Table 4. Achievement per Test Item

No Items	Score Average	Percentage
1	2,2	54%
2	2,4	59%
3	2,2	55%
4	2,7	67%
5	1,9	47%
6	2,0	50%

Based on Table 4, Item 1 required students to interpret and coordinate visual representations (area models), symbolic representations (fractions and probability multiplication), and verbal representations (explanations of the relationship between the diagram and the computation) in determining the probability of compound events and their complementary events. The achievement level of 54% indicates that many students still experienced difficulties in the progressive formalization process, particularly when transforming visual models into symbolic computations and explaining their reasoning conceptually.



Translated into English

1. a. Sonia (probability = $\frac{2}{5}$)
   Aysa (probability = $\frac{1}{3}$)
   The relationship between Sonia's diagram and Aysa's probability of coming The possibility: each diagram shows how the probability is obtained. The relationship is the same area; overlapping probabilities.

b. The probability that Sonia and Aysa both come?
   $\frac{2}{5}$ x $\frac{1}{3} = \frac{2}{15}$
   Meaning: out of 15 possibilities, only 2 allow both of them to come.

c. The probability that not a single one of them comes?
   Sonia does not come → probability: $1 - \frac{2}{5} = \frac{3}{5}$
   Aysa does not come → probability: $1 - \frac{1}{3} = \frac{2}{3}$
   $\frac{3}{5}$ x $\frac{2}{3} = \frac{6}{15} = \frac{2}{3}$
   (Probability that they do not come)

Figure 2. Example of Students' Work for Item 1

Based on Figure 2, the student's response to Item 1 illustrates an attempt to explain the probability of compound events using an area model while coordinating visual, symbolic, and verbal representations. Visually, the student drew two rectangular area models labeled "Sonia ($\frac{2}{5}$)" and "Aysa ($\frac{1}{3}$)," each divided into equal parts to represent their respective probabilities. In the written explanation next to the diagrams (line 2), the student stated, "*setiap diagram itu menunjukan bagaimana kedua peluang itu ketemu di area yang sama, tumpang tindih peluang*" (S1, translated into English: "each diagram shows how the two probabilities meet in the same area, overlapping probabilities"). This statement suggests that the student associated the area representation with the simultaneous occurrence of the two events, although the explanation remains informal. This finding aligns with the graphical results indicating that visual representation achieved the highest score (60%), as visual models consistently served as the starting point for understanding probabilistic situations.

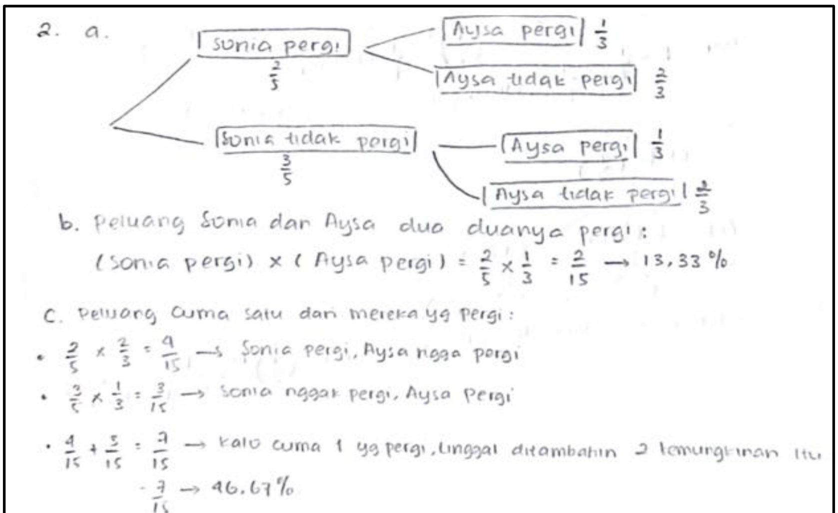
The student then proceeded to express the relationship symbolically by writing "$\frac{2}{5}$× $\frac{1}{3}$= $\frac{2}{15}$" (line 4), accompanied by the explanation "*artinya: dari 15 kemungkinan, cuman 2 yg bisa bikin mereka berdua datang*" (S1; translated into English: "this means that out of 15 possibilities, only 2 make both of them come"). This step indicates a shift from the visual representation toward symbolic computation. However, the written explanation does not explicitly connect the multiplication operation to the overlapping region represented in the diagram, suggesting that the symbolic procedure may not be fully grounded in the visual model. Nevertheless, as indicated by the symbolic representation achievement score (54%), the use of mathematical symbols remained predominantly procedural and was not fully reconnected to the meaning of the overlapping area in the visual model.

In addition, the student used complementary reasoning to determine the probability that neither Sonia nor Aysa would attend. In the lower part of the response (lines 6–8), the student wrote

"*Sonia ngga datang → peluang:* $1-\frac{2}{5}=\frac{3}{5}$" and "*Aysa ngga datang → peluang:* $1-\frac{1}{3}=\frac{2}{3}$" (S1; translated into English: "Sonia does not attend → probability: $1-\frac{2}{5}=\frac{3}{5}$" and Aysa does not attend → probability: $1-\frac{1}{3}=\frac{2}{3}$"), followed by the calculation "$\frac{3}{5}\times\frac{2}{3}=\frac{6}{15}=\frac{2}{5}$. While these statements demonstrate the use of symbolic procedures and written explanations, the student did not explicitly relate this reasoning to the initial visual representation. This is consistent with the graphical findings showing that verbal representation had the lowest achievement score (48%), indicating students' difficulties in explaining strategies, linking multiple representations, and reflecting on the outcomes of their calculations.

Overall, this response indicates that the student attempted to coordinate visual, symbolic, and verbal representations when solving the task. However, the written explanations mainly describe the computational steps without explicitly referring back to the overlapping regions in the area model or explaining how the visual representation supports the symbolic operations. As a result, the connections among the visual model, the multiplication procedure, and the complementary reasoning remain only partially articulated. This suggests that although the student was able to use multiple representations, difficulties remain in clearly expressing the relationships among them through written explanations.

Based on Table 4, Item 2 required students to coordinate visual representations (area models and probability trees), symbolic representations (multiplication, addition, fractions, and percentages), and verbal representations (process explanations) in determining the probability of compound events and "only one occurs" events. The medium-to-low achievement level of 59% indicates that although some students were able to perform probability computations, many still experienced difficulties in linking visual models with symbolic operations and articulating their reasoning verbally within the RME framework.



Translated into English

2. a. Sonia goes → $\frac{2}{5}$
- Aysa goes → $\frac{1}{3}$
- Aysa does not go → $\frac{2}{3}$

Sonia does not go → $\frac{3}{5}$
- Aysa goes → $\frac{1}{3}$
- Aysa does not go → $\frac{2}{3}$

b. Probability that Sonia and Aysa both go
(Sonia goes)×(Aysa goes) = $\frac{2}{5}$ x $\frac{1}{3}=\frac{2}{15}=13{,}33\%$

c. Probability that only one of them goes
- $\frac{2}{5}$ x $\frac{1}{3}=\frac{4}{15}$ → Sonia goes, Aysa does not go
- $\frac{3}{5}$ x $\frac{1}{3}=\frac{3}{15}$ → Sonia does not go, Aysa goes

$\frac{4}{15}+\frac{3}{15}=\frac{7}{15}$

$\frac{7}{15}=46{,}67\%$

Figure 3. Example of the Students' Work for Item 2

Based on Figure 3, students' responses to Item 2 showed the use of probability trees to determine the probability of compound events and the combined "only one occurs" event. Visually, students were able to complete the structure of the probability tree with the branches "Sonia goes/does not go" and "Aysa goes/does not go," along with the corresponding probabilities. This visual model functioned as a model-of the contextual situation, facilitating students in organizing all possible events systematically. This finding aligns with the graphical results showing that visual representation achieved the highest score (60%), as the probability tree explicitly helped students map the sample space of compound events.

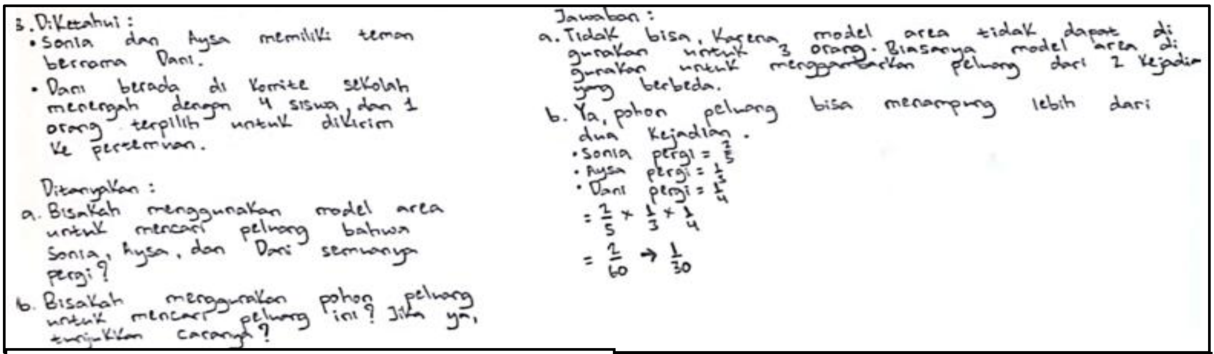
Subsequently, the students used the probability tree as a model-for in performing symbolic computations. This was evident from their use of fractional multiplication to determine the probability that both students would go, as well as the addition of probabilities for the combined "only one goes" event. This process reflects the stage of progressive formalization, in which students transformed the visual model into more formal mathematical procedures. Nevertheless, as indicated by the symbolic representation achievement score (54%), some students still performed the computations mechanically without explicitly reconnecting them to the meaning of each branch in the probability tree.

In terms of verbal representation, students provided brief written explanations of their strategies, for instance by stating that the probability of "only one goes" was obtained by adding the two relevant cases. Although this strategy was mathematically correct, the explanation did not fully reflect a process of reflection on the result, as students did not specify why the two events were added nor how the final outcome related to the context of the problem. This condition is consistent with the graphical findings showing that verbal representation had the lowest achievement score (48%), indicating students' limited ability to communicate relationships among representations and to carry out conceptual reflection as required in RME-based instruction.

Thus, the analysis of Figure 2 supports the findings in Table 3 that Item 2 required the coordination of visual representations (probability trees), symbolic representations (multiplication, addition, fractions, and percentages), and verbal representations (process explanations). The achievement level of 59% indicates that although some students were able to compute probabilities correctly, many still experienced difficulties in linking visual models with symbolic operations and articulating their reasoning verbally within the RME framework.

Based on Table 4, Item 3 required students to reason about compound probabilities involving more than two individuals while simultaneously evaluating the appropriateness of representation models (area models and probability trees) through logical verbal explanations. The achievement level of 55% indicates that many students still experienced difficulties in reflecting on the limitations and usefulness of the models and in linking the problem context with the appropriate mathematical representations within the RME framework.

Based on Figure 4, students' responses to Item 3 demonstrated attempts to reason about compound probabilities involving more than two individuals (Sonia, Aysa, and Dani) while also evaluating the suitability of representation models. In part (a), students verbally stated that the area model was not appropriate for representing the probability of three individuals due to its limitations in depicting sequential branching of events. This statement indicates that students had conceptually understood the function of visual representations, positioning the area model as a model-of that is not necessarily relevant for all probabilistic situations. This ability reflects a process of reflection on model limitations, which constitutes an important characteristic of the RME approach.


3. Diketahui :
• Sonia dan Aysa memiliki teman bersama Dani.
• Dani berada di Komite sekolah menengah dengan 4 siswa, dan 1 orang terpilih untuk dikirim ke pertemuan.

Ditanyakan :
a. Bisakah menggunakan model area untuk mencari peluang bahwa Sonia, Aysa, dan Dani semuanya pergi?
b. Bisakah menggunakan pohon peluang untuk mencari peluang ini? Jika ya, tunjukkan caranya?

Jawaban :
a. Tidak bisa, karena model area tidak dapat digunakan untuk 3 orang. Biasanya model area digunakan untuk menggambarkan peluang dari 2 kejadian yang berbeda.
b. Ya, pohon peluang bisa menampung lebih dari dua kejadian.
• Sonia pergi = $\frac{2}{5}$
• Aysa pergi = $\frac{1}{3}$
• Dani pergi = $\frac{1}{4}$
$= \frac{2}{5} + \frac{1}{3} + \frac{1}{4}$
$= \frac{2}{60} \rightarrow \frac{1}{30}$

Translated into English
Given:
- Sonia and Aysa have a friend named **Dani**.
- They are members of the **school committee**.
- In the school committee meeting, there are **4 students**, and **1 person is selected** to attend the meeting.

Questions:
a. Can the **area model** be used to determine the probability that **Sonia, Aysa, and Dani all attend the meeting**?
b. Can the **probability tree diagram** be used to determine this probability? If yes, **explain how**.

Translated into English
Answer:
a. Cannot, because the **area model cannot be used to represent three people**. Usually, the area model is used to represent probabilities of **two different events**.
b. Yes, a **probability tree** can accommodate **more than two events**.
Sonia goes = $\frac{2}{5}$, Aysa goes = $\frac{1}{3}$, Dani goes = $\frac{1}{4}$

$\frac{2}{5} + \frac{1}{3} + \frac{1}{4}, \frac{2}{60} \rightarrow \frac{1}{30}$

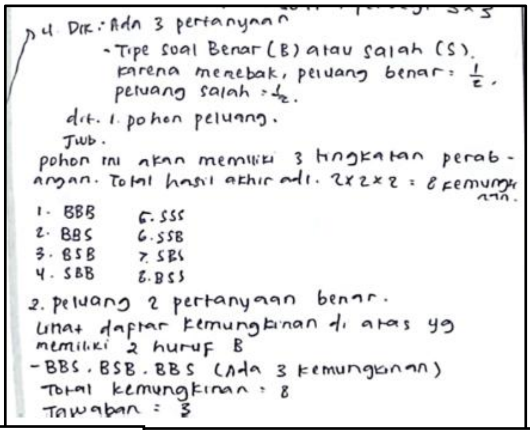
Figure 4. Example of the Students' Work for Item 3

In part (b), students shifted to using a probability tree diagram as an alternative visual representation. This tree diagram functioned as a model-for in reasoning about compound probabilities with more than two stages of events. Students then assigned probabilities to each branch and combined them symbolically through multiplication and addition to determine the required event probability. This process demonstrates intertwinement among representations, in which visual (tree diagram), symbolic (probability fractions on each branch), and verbal (explanations of model selection) representations were interconnected within a single chain of reasoning.

However, although students were able to carry out symbolic computations and select a more appropriate representation model, their verbal explanations remained brief and did not fully connect the computational results to the problem context in a meaningful manner. This is consistent with the graphical findings showing that verbal representation had the lowest achievement score (48%), while symbolic representation reached 54%. Students tended to focus on the numerical outcomes without providing conceptual elaboration regarding the meaning of the obtained probabilities or their implications for the contextual situation.

These findings support Table 3, which shows that Item 3 had an achievement level of 55%. This relatively low level indicates that although some students were able to recognize the limitations of the area model and shift to the probability tree diagram, many still experienced difficulties in comprehensively reflecting on the usefulness and constraints of the models and in verbally communicating their reasons for selecting specific representations.

Based on Table 4, Item 4 required students to construct and use a probability tree as a visual representation and to coordinate it with symbolic computations (multiplication, addition, and complementary probability) and verbal explanations when determining compound events. The achievement level of 67% indicates that most students were able to follow the computation process supported by the visual model, although limitations remained in their interpretation of relationships among events and in their conceptual reflection within the RME framework.


4. Dik : Ada 3 pertanyaan
- Tipe soal Benar (B) atau salah (S).
karena menebak, peluang benar : $\frac{1}{2}$.
peluang salah : $\frac{1}{2}$.
dit. 1. pohon peluang.
Jwb.
pohon ini akan memiliki 3 tingkatan percabangan. Total hasil akhir adl. 2×2×2 = 8 kemungkinan.
1. BBB 5. SSS
2. BBS 6. SSB
3. BSB 7. SBS
4. SBB 8. BSS
2. peluang 2 pertanyaan benar.
Lihat daftar kemungkinan di atas yg memiliki 2 huruf B
- BBS . BSB . BBS (Ada 3 kemungkinan)
Total kemungkinan : 8
Jawaban : 3

Translated into English **Given:** There are 3 questions. – Type of questions: True (B) or False (S). Because the student guesses, the probability of a correct answer is $\frac{1}{2}$, and the probability of an incorrect answer is $\frac{1}{2}$. **Asked:** Determine the probability.	**Solution:** This probability tree has 3 stages of branching. The total number of possible outcomes is obtained from 2×2×2=8 possible outcomes. 1.BBB, 2. BBS, 3.BSB, 4. SBB, 5. SSS, 6. SSB, 7. SBS, 8. BSS **Probability that 2 questions are answered correctly** Observe the list of possible outcomes above that contain two letters **B**: – BBS, BSB, SBB (there are 3 possible outcomes) Total possible outcomes: 8 **Answer:** $\frac{3}{8}$

Figure 5. Example of the Students' Work for Item 4

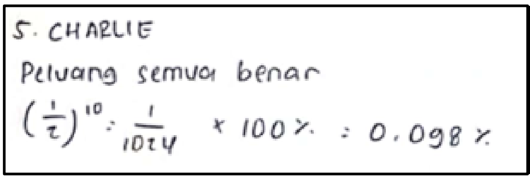
Based on Figure 5, the students' responses to Item 4 showed their ability to compute probabilities in repeated trials involving two possible outcomes, namely correct (B) and incorrect (S), conducted three times. Students began the solution by constructing a probability tree as a visual representation to map all possible outcomes of the experiment. The use of the probability tree functioned as a model-of the repeated experiment situation, as it helped students systematically organize the sample space through branching at each stage of the trial.

Subsequently, the probability tree was used as a model-for performing symbolic probability computations. Students explicitly stated that there were $2 \times 2 \times 2 = 8$ possible outcomes, and then identified the event satisfying the condition "two correct answers" by counting the relevant arrangements (BBB, BBS, BSB). Some responses reflected procedural enumeration, where students simply listed outcome sequences without explaining how these sequences correspond to branches of the tree. Other responses indicated emerging structural reasoning, in which students explicitly related each sequence to a path in the probability tree and recognized that the probability of a sequence is obtained by multiplying the probabilities along the corresponding branches. Indicators of this structural reasoning included references to branch probabilities and explanations that connected each event sequence to its position in the tree structure. This process reflects progressive formalization, in which students moved from the visual model to symbolic computation using the concepts of probability multiplication and fractions. The probability was then determined by comparing the number of favorable outcomes to the total sample space, which was subsequently expressed in fractional form and simplified.

However, although most students' computational procedures were correct, their verbal explanations largely remained procedural, focusing on counting possible outcomes and presenting the final fraction without elaborating on why branch probabilities are multiplied or how independence of trials justifies the tree structure. This finding is consistent with the graphical results showing that verbal representation had the lowest achievement score (48%), compared with visual representation (60%) and symbolic representation (54%). In other words, students were relatively strong in constructing and using visual models and fairly capable in symbolic manipulation, but still experienced difficulties in articulating structural reasoning about independence and the mapping of events to branches in the probability tree.

These findings support Table 4, which shows that Item 4 had an achievement level of 67%. This percentage indicates that most students were able to follow the probability computation process supported by the probability tree model in a systematic manner. Nevertheless, the remaining gap indicates that students' reasoning was often dominated by procedural enumeration rather than explicit structural interpretation of the tree, particularly in explaining how event sequences correspond to branches and how independence justifies the multiplication of probabilities in repeated trials.

Based on Table 4, Item 5 required students to understand and generalize the structure of probability in repeated trials with a large number of repetitions through a conceptual-visual description of a probability tree, symbolic computations based on probability multiplication, and verbal explanations justifying the strategy used. The achievement level of 47% indicates that many students still experienced difficulties in abstracting the pattern of repeated trials without full visual support and in explaining their reasoning conceptually, particularly during the progressive formalization process within the RME framework.



5. CHARLIE
Probability that all answers are correct
$\left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \text{x } 100\% = 0{,}098\%$

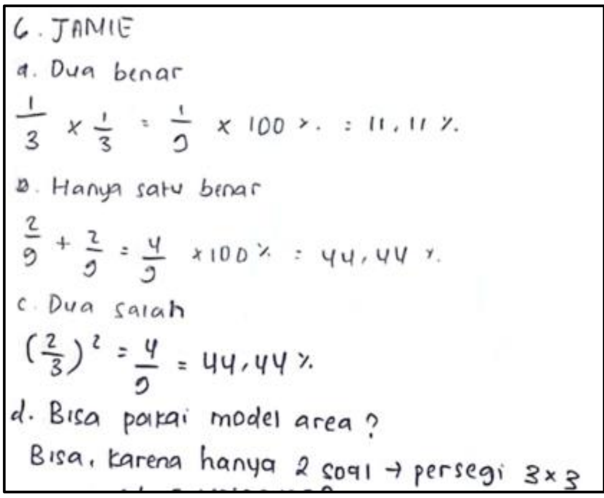
Figure 6. Example of Students' Work for Item 5

Based on Figure 6, the students' responses to Item 5 showed attempts to determine the probability of repeated trials with a large number of repetitions, specifically the probability that all outcomes are correct. The students directly wrote the symbolic form of the probability as $\left(\frac{1}{2}\right)^{10}$, which was then converted into percentages. This approach indicates that students had recognized the general structure of repeated trials and were able to perform symbolic generalization through the use of probability exponentiation, which represents the result of repeatedly multiplying identical probabilities. In this context, the idea of the probability tree was present implicitly as a conceptual model-of, even though it was not explicitly drawn in a visual form.

Then, the expression $\left(\frac{1}{2}\right)^{10}$ functioned as a model-for in the formal probability computation. The transformation from repeated multiplication to exponentiation reflects the stage of progressive formalization, in which students moved from concrete modeling toward a more abstract and efficient mathematical representation. However, although the symbolic computation was performed correctly, the students' responses did not include verbal justification regarding the reason for using exponentiation, its relation to the concept of repeated trials, nor the meaning of the very small probability obtained within the context of the problem.

This condition is consistent with the graphical results indicating that symbolic representation reached 54%, whereas verbal representation had the lowest achievement score (48%). Students appeared to be capable of procedural symbolic manipulation but were not yet fully able to articulate the conceptual reasoning underlying their strategies. In addition, the absence of explicit visual representations, such as probability trees or hierarchical schemes, suggests that some students experienced difficulties in abstracting the structure of repeated trials without full visual support. These findings support Table 4, which shows that Item 5 had an achievement level of 47%. This relatively low level indicates that many students were not yet fully able to generalize the probability structure of repeated trials with large sample spaces and explain their reasoning verbally within the RME framework.

Based on Table 4, Item 6 required students to model a two-stage probability experiment with more than two possible outcomes, compute the probability of single and compound events using probability trees, multiplication, addition, and complementary probability, while simultaneously explaining and reflecting on the strategies used. The achievement level of 50% indicates that some students were able to perform basic computations and interpret the probability structure, but still experienced difficulties in consistently integrating visual, symbolic, and verbal representations and in conducting full model reflection in accordance with RME principles.


6. JAMIE
a. Dua benar
$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \times 100\% = 11,11\%$
b. Hanya satu benar
$\frac{2}{9} + \frac{2}{9} = \frac{4}{9} \times 100\% = 44,44\%$
c. Dua salah
$(\frac{2}{3})^2 = \frac{4}{9} = 44,44\%$
d. Bisa pakai model area ?
Bisa, karena hanya 2 soal → persegi 3x3

6. JAMIE
a. Two correct answers
$\frac{1}{3}\text{x}\frac{1}{3} = \frac{1}{9}$ x 100% = 11,11%

b. Only one correct answer
$\frac{2}{9} + \frac{2}{9} = \frac{4}{9}$ x 100% = 44,44%

c. Two incorrect answers
$(\frac{2}{3})^2 = \frac{4}{9}$ = 44,44%

d. Can the area model be used?
Yes, it can, because there are only two questions, so a 3 × 3 square can be used.

Figure 7. Example of Students' Work for Item 6

Based on Figure 7, it can be seen that students had a general understanding of probability in repeated trials with more than two possible outcomes, although their understanding remained partial and tended to be procedural. In part (a), students computed the probability of obtaining two correct answers by multiplying $\frac{1}{3}\text{x}\frac{1}{3} = \frac{1}{9}$, which indicates mastery of symbolic representation in the form of probability multiplication for compound events and an understanding that the two questions were independent. However, the use of $\frac{1}{3}\text{x}\frac{1}{3} = \frac{1}{9}$ x 100% = 11,11% was not appropriate. This indicates that the probability tree as a visual representation had implicitly functioned as a model-of the situation, even though it was not drawn explicitly, and that the answer produced was not accurate.

In part (b), the students determined the probability of "only one correct answer" by adding probabilities of $\frac{2}{9} + \frac{2}{9} = \frac{4}{9}$. This step shows that the students had recognized the presence of two mutually exclusive events (correct–incorrect and incorrect–correct), although the verbal explanation regarding the reason for adding the probabilities remained limited and did not fully connect the calculated result to the structure of the probability tree. Besides, the use of $\frac{2}{9} + \frac{2}{9} = \frac{4}{9}$ x 100% = 44,44% was not appropriate enough. This condition is consistent with the achievement level of 50%, which indicates that students were able to perform basic computations, but were not yet strong in coordinating visual, symbolic, and verbal representations in a comprehensive manner.

In part (c), the students used complementary probability by writing the probability of "two incorrect answers" as $(\frac{2}{3})^2 = \frac{4}{9}$. The use of this method indicates an alternative and more efficient form of understanding, although it was presented briefly and without deeper conceptual reflection.

The use of $(\frac{2}{3})^2 = \frac{4}{9} = 44{,}44\%$ was also appropriate enough. Meanwhile, in part (d), students stated that the area model could be used because the task involved only two questions and referred to a "3×3 square" representation. This statement indicates an awareness of the possibility of using alternative models, but the reflection on how the area model would function as a model-for remained descriptive and was not explicitly connected to the meaning of probability.

These findings are consistent with the graphical results showing that visual representation had the highest achievement score (60%), followed by symbolic representation (54%), while verbal representation had the lowest (48%). This pattern reinforces that students were relatively strong in reading and utilizing probability structures and performing symbolic manipulation, but still experienced difficulties in consistently integrating visual, symbolic, and verbal representations and in reflecting on model use in accordance with RME principles. Accordingly, students' written work supports Table 4, which shows that Item 6 had an achievement level of 50%. This percentage indicates that some students were able to complete basic probability computations and understand the structure of two-stage experiments with more than two possible outcomes. However, the less-than-optimal achievement suggests that conceptual reflection and mathematical communication still require strengthening so that students are not only able to compute probabilities, but also able to explain and evaluate representational use more fully within the framework of RME-based instruction.

Discussion

In solving MiC problems on computing chances, students interacted with three forms of mathematical representation: visual, symbolic, and verbal. The results of this study show that visual representation achieved the highest performance (60%), followed by symbolic representation (54%), while verbal representation was the lowest (48%). This is in line with the results of study by Puteri and Priatna (2024) indicating that the students tended to begin problem solving with visual representations before moving to symbolic and then verbal representations, an order that is consistent with the principle of progressive formalization in RME in which visual understanding functions as a contextual model-of before being formalized into symbolic representations as a model-for. This is also in line with the study by Rif'at et al., (2024), stating that visual representations served as the dominant foundation for students' initial conceptual representation in mathematics learning, while other representations developed only after visual understanding had been established. This finding implies that visual representation functions as an initial foundation for conceptual understanding, whereas symbolic representation serves as the formal expression of mathematical reasoning constructed by students. The substantial gap between visual/symbolic and verbal performance is consistent with previous studies that have examined patterns of students' mathematical representation abilities across various mathematical topics, in which visual and symbolic representations tend to be more developed than verbal representations. The results by (Imama & Caswita, 2023; Ünal et al., 2023; Žakelj & Klančar, 2022) analyzing the students' mathematical representation abilities across various mathematical contexts have found that visual and symbolic skills are often stronger, whereas verbal representation tends to be the weakest indicator because students struggle to articulate their thinking conceptually. Beyond confirming this pattern, the present study contributes more specifically by showing that MiC tasks effectively support the stability of visual models as model-of representations, yet they do not automatically promote a bridging process toward verbal explanation. This indicates that the transition from visual–symbolic reasoning to verbal articulation remains a critical gap in the representational chain of students' probability reasoning.

Visual representations in solving MiC problems on computing chances facilitated students in mapping sample spaces, identifying combinations of events, and perceiving part–whole relationships directly through area diagrams and probability trees. The consistency of visual use is reinforced by studies on probability learning (Batanero & Álvarez-Arroyo, 2024), which emphasize that visual representations help students overcome difficulties in understanding the abstract nature of probability concepts and in identifying patterns in complex compound events, thereby making reasoning more transparent. In addition, these studies highlight the importance of early visual experiences in probability instruction for building students' conceptual frameworks. A meta-analytic literature review further confirms that the use of visualization in mathematics instruction can improve learning outcomes with a medium effect size across various mathematical topics, including chance and probability concepts (Schoenherr et al., 2024). The consistency of visual representation use is also in line with the results of study by Sariningsih and Herdiman (2017), showing that visualization plays a key role in developing statistical reasoning and creative mathematical thinking among students. Although conducted at a different educational level, these findings indicate that visual and statistical representations function as initial tools for constructing conceptual meaning before being formalized into symbolic forms, including in the context of chance and probability. More importantly, the present findings suggest that visual models in MiC tasks function not only as representational aids but also as cognitive organizers that stabilize students' reasoning structures when dealing with compound events.

Symbolic representation emerged when students transformed visual models into formal notation, namely probability fractions, multiplication, addition, complementary probability, and exponentiation in repeated trials. The finding that symbolic representation achieved 54% indicates that students procedurally understood how to compute probabilities but were not yet able to relate symbols back to the underlying visual functions. This limitation is consistent with empirical evidence showing that symbolic representational competence must be developed through strong contextual integration so that students do not merely "calculate" but also "understand" the meaning of symbols in probability contexts (Puteri & Priatna, 2024). This pattern suggests that progressive formalization in the learning process tended to stall at the stage of procedural formalization, where symbols are used correctly for computation but are not fully reconnected to the meanings embodied in the visual models from which they originated.

Verbal representation achieved 48%, indicating that students experienced difficulties in explaining the relationships between representations and in communicating their reasoning strategies conceptually. Yet, verbal mathematical skill is an important indicator of conceptual mastery because it enables students to explain why and how their strategies work, rather than merely producing correct answers. This is consistent with findings by Žakelj & Klančar (2022), which emphasizes that verbal activities such as explaining solution strategies, formulating reasons for model selection, and reflecting on understanding can enhance the integration among representations and strengthen students' conceptual understanding. The relatively low verbal performance therefore signals that the reflective stage of progressive formalization, where students reconnect meaning, representation, and explanation, has not yet been fully realized in students' reasoning processes.

These findings indicate that the students possessed a reasonably adequate foundational ability in visual and symbolic aspects; however, the suboptimal performance in verbal representation indicates the need for further reinforcement in conceptual reflection and mathematical communication. Such reinforcement is necessary not only so that students can compute probabilities, but also so that they can explain and evaluate the use of multiple representations holistically within the framework of RME-based instruction. In this regard, the study highlights the importance of designing learning tasks that explicitly require students to translate visual models into written explanations and interpret symbolic expressions in contextual terms, thereby strengthening the representational bridge that currently remains weak.

This study not only identified the types of mathematical representations used by the students, but also revealed how intertwinement was constructed through their mathematical thinking processes. Within the RME framework, such interconnection is a key principle because visual, symbolic, and verbal representations are not viewed as independent entities, but must be intertwined and mutually reinforcing within a meaningful chain of mathematical reasoning. Intertwinement allows students to move flexibly across representations so that models that initially function as model-of contextual situations can develop into model-for more formal mathematical reasoning. These findings are consistent with studies that emphasize that the quality of students' mathematical understanding is strongly determined by their ability to connect and translate among multiple forms of representation in a consistent manner, particularly in probability concepts that are abstract and hierarchical in nature (Batanero & Álvarez-Arroyo, 2024; Jeannotte & Kieran, 2017). The present analysis therefore contributes by illustrating how the representational chain in probability learning is not only hierarchical but also fragile, particularly at the stage where conceptual meaning must be articulated verbally.

In this study, visual representation served as the foundational layer in constructing students' understanding. The use of area models and probability tree diagrams helped students construct sample spaces and event relations in a concrete and structured manner. In Items 1 and 2, students first mapped all possible events before performing formal operations, indicating that visualization functioned as an initial thinking tool for identifying patterns and event structures. The ability to detect structure through these diagrams facilitated students' understanding of compound event relationships, which subsequently served as a basis for formalization. These results are consistent with research in probability learning showing that visualization is an important means of bridging students' intuitive understanding with formal probability concepts, especially in multi-step experiments and conditional events (Batanero & Álvarez-Arroyo, 2024). This indicates that tasks that explicitly present structural visual models, such as probability trees, are more likely to trigger representational intertwinement because they make the relational structure of events visible to students. This suggests that instructional design must deliberately reintroduce visual meaning during symbolic manipulation so that formal procedures remain connected to conceptual structures.

After visual understanding was established, students transformed it into symbolic representation as part of the progressive formalization process. This stage reflects a shift from situational understanding toward mathematical abstraction, in which operations such as probability multiplication, addition, and complementary probability emerged as formalizations of previously constructed visual structures. However, the findings indicate that the connection between visual and symbolic representations was not fully robust. Students were able to perform symbolic computations procedurally, but did not explicitly relate the symbols they used to the underlying visual meaning. This condition is consistent with studies showing that students often become trapped in symbolic manipulation without conceptual reflection when transitions between representations are not explicitly facilitated in instruction (Ario et al., 2025; Tuveri et al., 2026; Puteri & Priatna, 2024).

Verbal representation is expected to function as a conceptual binder that connects visual meaning with symbolic expression through explanation, argumentation, and reflection. Through verbal representation, students are expected to justify model selection, interpret the symbols used, and relate solution steps to the context of the problem. However, the low performance in verbal representation indicates that this reflective and communicative dimension has not yet developed optimally. Research on mathematical representation has shown that without instructional scaffolding that deliberately encourages students to articulate their reasoning orally or in writing, the connections among representations tend to remain fragmented and superficial (Jeannotte & Kieran, 2017; Riccomini et al., 2024). In the context of RME-based instruction, the use of scaffolding has been shown to help students transform visual representations into symbolic and verbal forms more effectively, supporting the principle of progressive formalization that emphasizes interconnection among representations (Putra et al., 2024). Consequently, verbal explanation should be positioned not merely as a reporting activity but as a cognitive process that consolidates meaning across representations.

This configuration of representational interconnection was more evident in Item 4, which achieved a score of 67%. In this item, students followed a more consistent solution path: they constructed a probability tree diagram visually and then carried out symbolic computations based on the structure of the diagram. Although students' verbal explanations remained predominantly procedural, these results indicate that clear visual structure and organized computational tasks can facilitate better integration across representations. This is in line with the study by Amadeus et al., (2025) that the use of visual instructional media has been shown to improve students' mathematical skills, reaffirming the role of visual representation as a foundational layer before the transition to symbolic and verbal forms. These findings reinforce the view that the quality of contextual problem design in RME plays a crucial role in fostering representational intertwinement, although strengthening the verbal dimension is still required. In particular, task types that provide explicit structural representations, such as tree diagrams, appear to trigger stronger coordination between visual and symbolic reasoning, suggesting a promising design principle for future MiC-based probability tasks.

Overall, the interconnection among representations in the topic of probability shows that visual representation serves as the primary foundation, symbolic representation enhances precision and computational efficiency, and verbal representation facilitates conceptual reflection and mathematical communication. These three dimensions need to be developed simultaneously and continuously through RME-based instructional design that emphasizes translation and reflection across representations. Taken together, the findings contribute to the literature by clarifying how MiC-based tasks support the early stages of representational construction while also revealing the critical point where progressive formalization tends to weaken, namely in the reconnection of meaning through verbal explanation.

Implication

The findings of this study have implications for RME-based instruction in probability, particularly in developing students' mathematical representation skills. The dominance of visual representation suggests that visual models such as area models and probability trees are effective as foundational models (model-of) for helping students understand sample space structures and relationships among events. Therefore, teachers should deliberately maintain the use of visual representations as the starting point of probability instruction, especially in computing chances tasks that are abstract and sequential in nature. More specifically, the results indicate that instructional prompts should be embedded immediately after students complete a visual model (e.g., after finishing a probability tree or area diagram). At this stage, teachers can ask targeted questions such as "What does each branch represent?", "Which events correspond to these paths?", or "How many outcomes are represented in this structure?" These prompts help students verbalize the meaning of the visual structure before moving to symbolic procedures, thereby stabilizing the model-of stage of reasoning.

However, the moderate performance in symbolic representation and the relatively low performance in verbal representation indicate that the process of progressive formalization has not yet been fully optimized. The pedagogical implication is that instructional design must not only facilitate the transition from visual to symbolic forms, but also explicitly emphasize the reconnection of symbolic meaning to the underlying visual models from which it emerged. Teachers need to provide instructional scaffolding that encourages students to justify their use of mathematical operations, articulate the meaning of the resulting probabilities, and explain relationships among events at a conceptual level. Mechanism based scaffolding can be implemented at the moment when students are about to translate visual structures into symbolic expressions, for example, immediately before performing symbolic multiplication or addition of probabilities. At this stage, teachers can prompt students with questions such as "Which branches of the tree correspond to this multiplication?", "Why do these probabilities need to be multiplied rather than added?", or "How does this calculation relate to the path structure in your diagram?" Such prompts explicitly link symbolic operations to their visual origins, strengthening the conceptual transition from model-of to model-for.

The low performance in verbal representation underscores the importance of strengthening mathematical reflection and communication within RME-based instruction. Activities such as structured classroom discussion, short reflective writing, and prompting questions that require students to answer "why" and "how" should be systematically integrated into probability instruction. Through such pedagogical strategies, verbal representation can function as a conceptual binder that unifies visual and symbolic representations into a coherent chain of mathematical reasoning. In practical terms, reflective prompts can be positioned after students obtain their final probability results. For instance, teachers may ask students to briefly explain in writing or discussion prompts such as "What does this probability mean in the context of the problem?", "How did the diagram help you reach this result?", or "Could this result be obtained in another way?" Placing reflection at the end of the solution process encourages students to reinterpret their symbolic results through visual and contextual meaning, thereby strengthening verbal articulation of reasoning.

Theoretically, these findings reinforce the principle of intertwinement within RME, namely that the quality of students' mathematical understanding is largely determined by their ability to coordinate multiple representations rather than merely mastering a single form in isolation. Therefore, MiC task design needs to continue to be developed with a balanced emphasis on modeling, formalization processes, and verbal reflection so that probability instruction does not become confined to procedural execution alone. In particular, the results suggest that MiC task sequences should deliberately incorporate three critical representational checkpoints: (1) a meaning-construction phase immediately after visual modeling, (2) a meaning-connection phase before symbolic computation, and (3) a reflection phase after obtaining the final probability. Embedding prompts at these specific stages can operationalize the principle of intertwinement by ensuring that visual, symbolic, and verbal representations interact continuously throughout the problem-solving process.

Limitation and Suggestion for Further Research

This study has several limitations that should be acknowledged. First, the number of participants was relatively limited and drawn from a single class in one school, which restricts the generalizability of the findings. Future studies are recommended to involve larger and more diverse samples, both in terms of school characteristics and student backgrounds, in order to obtain a more comprehensive picture of students' mathematical representation abilities in probability. In addition, although students' responses were evaluated using predetermined scoring criteria, the study did not formally report inter-rater reliability measures. Future studies should therefore include procedures such as independent double scoring and statistical agreement indices to strengthen the reliability of the interpretation of students' representational performance.

Second, this study focused on analyzing students' written work on MiC tasks, meaning that students' representational thinking processes were largely inferred from written products. Future research could combine written analysis with other qualitative data, such as in-depth interviews or think-aloud protocols, to reveal students' intertwinement processes and conceptual reflection more thoroughly. Furthermore, the MiC tasks used in this study were adapted to fit the instructional context and language of the students.

Third, this study did not specifically explore the role of scaffolding strategies or particular instructional interventions in improving verbal representation. Future research may therefore investigate the effectiveness of RME-based instructional designs that explicitly target mathematical communication, for example through guided reflection, written justification tasks, or argumentation-based classroom discussions. Another limitation concerns the fidelity of the RME-based instructional implementation. The study did not systematically document aspects such as instructional duration, the extent to which the teacher followed the intended MiC learning trajectory, or the specific facilitation strategies used during classroom interaction. Variations in teacher mediation and classroom enactment may influence how students engage with visual models, symbolic procedures, and verbal explanation. Future studies should therefore monitor implementation fidelity more explicitly, including documentation of lesson duration, teacher roles in facilitating modeling and discussion, and the consistency of RME principles applied throughout the instructional process.

CONCLUSION

This study aimed to examine the forms of mathematical representation used by students and the interconnections among these representations in solving Mathematics in Context (MiC) tasks on computing chances within the framework of Realistic Mathematics Education (RME). The findings indicate that students employed three forms of representation: visual, symbolic, and verbal, with different levels of performance. Visual representation was the most dominant, particularly through area models and probability tree diagrams that helped students structure sample spaces and identify relationships among events, while symbolic representation was used mainly to perform probability computations. In contrast, verbal representation showed the lowest performance, indicating that students experienced difficulties in explaining their reasoning and articulating the meaning of the obtained probabilities.

These findings contribute to the literature by showing that MiC tasks in an RME context effectively support the early stages of representational reasoning through visual modeling, but the transition toward fully integrated representational understanding remains incomplete when verbal articulation is not sufficiently supported. A clear direction for future research is therefore the development and empirical testing of mechanism-based verbal scaffolds positioned at key stages of the solution process, such as after completing a probability tree, before performing symbolic multiplication or addition, and after obtaining the final probability result to strengthen the integration of visual, symbolic, and verbal representations in probability learning.

AUTHOR CONTRIBUTIONS STATEMENT

In this study, **RS** was responsible for conceptualisation, Writing-Original Draft, Formal analysis, and Methodology; and **N** focused on Review, Editing and Visualisation.

REFERENCES

Amadeus, M. V., Nurjanah, N., & Yulianti, K. (2025). Improving high school students' computational thinking ability and mathematical resilience with project-based learning assisted by GeoGebra. *Edumatica: Jurnal Pendidikan Matematika, 15*(1). https://doi.org/10.22437/edumatica.v15i1.42801

Arifah, K., Indrawatiningsih, N., & Afifah, A. (2020). Analisis kemampuan multiple representasi siswa dalam memecahkan masalah peluang. *JP2M (Jurnal Pendidikan dan Pembelajaran Matematika), 6*(2), 67–76. https://doi.org/10.29100/jp2m.v6i2.1749

Ario, M., Suhendra, Jupri, A., & Nurlaelah, E. (2025). Students' Errors and Learning Obstacles in Solving Algebraic Word Problems: Hermeneutic Phenomenology. *Education Sciences, 15*(12), 1674. https://doi.org/10.3390/educsci15121674

Awuah, F. K., & Ogbonnaya, U. I. (2020). Grade 12 students' proficiency in solving probability problems involving contingency tables and tree diagrams. *International Journal of Instruction, 13*(2), 819–834. https://doi.org/10.29333/iji.2020.13255a

Batanero, C., & Álvarez-Arroyo, R. (2024). Teaching and learning of probability. *ZDM–Mathematics Education, 56*(1), 5–17. https://doi.org/10.1007/s11858-023-01511-5

Batanero, C., & Carmen, D. (2012). Training school teachers to teach probability: Reflections and challenges. *Chilean Journal of Statistics, 3*(1), 3–13.

Batanero, C., Chernoff, E. J., Engel, J., Lee, H.-C., & Sánchez, E. (2016). *Research on teaching and learning probability*. Springer. https://doi.org/10.1007/978-3-319-31625-3

Creswell, J. W. (2016). *Research design: Pendekatan metode kualitatif, kuantitatif, dan campuran*.

Fadilah, N. S., & Hakim, D. D. L. (2022). Analisis kemampuan pemecahan masalah matematis siswa SMA pada materi fungsi. *Jurnal Theorems (The Original Research of Mathematics), 7*(1), 64. https://doi.org/10.31949/th.v7i1.3824

Freudenthal, H. (1991). *Revisiting mathematics education*. Kluwer Academic Publishers.

Garfield, J. B., & Ben-Zvi, D. (2008). *Developing students' statistical reasoning*. Springer.

Goldin, G. A. (2018). Mathematical representations. In *Encyclopedia of mathematics education*.

Goldin, G. A. (2020). Mathematical representations. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 566–572). Springer. https://doi.org/10.1007/978-3-030-15789-0_103

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*.

Gravemeijer, K. P. (1994). *Developing realistic mathematics education*. Technipress.

He, S. (2025). How do preservice mathematics teachers analyze and respond to student errors in solving probability problems using tree diagrams? *Teaching and Teacher Education, 168*, 105250. https://doi.org/10.1016/j.tate.2025.105250

Heuvel-panhuizen, M. Van den. (2003). The Didactical Use of Models in Realistic Mathematics Education: An Example From A Longitudinal Trajectory on Percentage. *Educational Studies in Mathematics, 54*, 9-35. https://doi.org/10.1023/B:EDUC.0000005212.03219.dc

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. University of Wisconsin Foundation.

Hitt, F. (2002). Representations and mathematics visualizations. In R. Speiser, C. A. Maher, & C. N. Walter (Eds.), *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (p. 53).

Holt, H., Rinehart, R., & Winston, W. (2006). *Second chance: Data analysis and probability*. Encyclopædia Britannica, Inc.

Imama, K., & Caswita. (2023). An analysis of mathematical representation ability of middle school students on concept congruence based on learning style. *Al-Jabar: Jurnal Pendidikan Matematika, 14*(1), 153–163. https://doi.org/10.24042/ajpm.v14i1.17320

Iversen, K., & Nilsson, P. (2019). Lower secondary school students' reasoning about compound probability in spinner tasks. *Journal of Mathematical Behavior, 56*, 100723. https://doi.org/10.1016/j.jmathb.2019.100723

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics, 96*(1), 1–16. https://doi.org/10.1007/s10649-017-9761-8

Jones, G. A., Langrall, C. W., & Thornton, C. A. (1995). A framework for assessing young children's thinking in probability. *Proceedings of the North American Chapter of the International Group for the Psychology of Mathematics Education*.

Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical Thinking and Learning, 5*(2–3), 157–189. https://doi.org/10.1080/10986065.2003.9679998

Marina, R., Zulkardi, Z., Susanti, E., & Meryansumayeka. (2025). Analisis kemampuan representasi matematis siswa SMP pada materi perbandingan menggunakan konteks jajanan. *Jurnal*

Pendidikan Matematika dan Sains, 13(1), 31–46. https://doi.org/10.21831/jpms.v13i1.79495

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representations in developing mathematical understanding. *Theory Into Practice, 40*(2), 118–127. https://doi.org/10.1207/s15430421tip4002_6

Post, M., & Prediger, S. (2024). Teaching practices for unfolding information and connecting multiple representations: the case of conditional probability information. *Mathematics Education Research Journal*, *36*(1), 97-129. https://doi.org/10.1007/s13394-022-00431-z

Puteri, A. R. K., & Priatna, N. (2024). Implementasi PBL dan RME pada materi statistika: Dampaknya terhadap kemampuan representasi matematis siswa. *Sigma Didaktika: Jurnal Pendidikan Matematika, 16*(2), 529–543. https://doi.org/10.26618/sigma.v16i2.15021

Putra, R. W. P., Sutiarso, S., & Nurhanurawati. (2024). Using the realistic mathematics education (RME) approach with scaffolding to enhance mathematical representation ability. *Al-Jabar: Jurnal Pendidikan Matematika, 15*(2), 535–546. https://doi.org/10.24042/ajpm.v15i2.24560

Riccomini, P. J., Hughes, E. M., Deshpande, D., Lee, J. Y., Fiveash, L., & Lin, T. H. (2024). Teaching fifth-grade students with specific learning disabilities to explain their mathematical reasoning through written expression. *Learning Disability Quarterly*, *47*(2), 124-136. https://doi.org/10.1177/07319487241235148

Rif'at, M., Sudiansyah, S., & Imama, K. (2024). Role of visual abilities in mathematics learning: An analysis of conceptual representation. *Al-Jabar: Jurnal Pendidikan Matematika, 15*(1), 87–97. https://doi.org/10.24042/ajpm.v15i1.22406

Rohmatin, D. N., Kurniati, D., & Yudianto, A. (2025, July). Visual Representation Profile of Junior High School in Solving The Geometric Word Problem. In *Education, Science, and Technology International Conference* (Vol. 3, No. 1, pp. 11-23).

Sariningsih, R., & Herdiman, I. (2017). Mengembangkan kemampuan penalaran statistik dan berpikir kreatif matematis mahasiswa melalui pendekatan open-ended. *Jurnal Riset Pendidikan Matematika, 4*(2), 239–246. https://doi.org/10.21831/jrpm.v4i2.16685

Schoenherr, J., Strohmaier, A. R., & Schukajlow, S. (2024). Learning with visualizations helps: A meta-analysis of visualization interventions in mathematics education. *Educational Research Review, 45*, 100639. https://doi.org/10.1016/j.edurev.2024.100639

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488. https://doi.org/10.3102/00028312033002455

Supriadi, A., & Ningsih, Y. L. (2022). Kemampuan representasi matematis mahasiswa pada materi distribusi peluang. *Indiktika: Jurnal Inovasi Pendidikan Matematika, 4*(2), 14–25. https://doi.org/10.31851/indiktika.v4i2.7678

Treffers, A. (1987). Integrated column arithmetic according to progressive schematisation. *Educational Studies in Mathematics, 18*, 125–145. https://doi.org/10.1007/BF00314723

Tuveri, M., Steri, A., & Fanti, V. (2026). Semiotic problem framing: a new framework to guide students and teachers in conceptual understanding and teaching of physics. *European Journal of Physics, 47*(1), 015712. https://doi.org/10.1088/1361-6404/ae2445

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2023). Visual and Symbolic Representations as Components of Algebraic Reasoning. *Journal of Numerical Cognition, 9*(2), 327-345. https://doi.org/10.5964/jnc.11151

Žakelj, A., & Klančar, A. (2022). The role of visual representations in geometry learning. *European Journal of Educational Research, 11*(3), 1393-1411. https://doi.org/10.12973/eu-jer.11.3.1393